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# PROCEEDINGS
*of*
# The Institute of Radio Engineers

EDITED BY
ALFRED N. GOLDSMITH, Ph.D.

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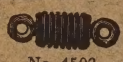

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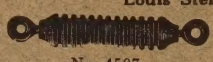

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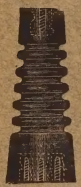

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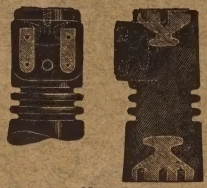

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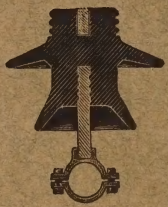

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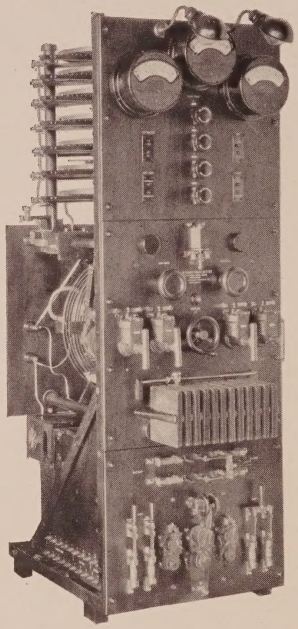

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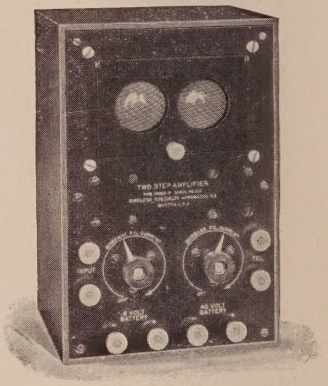

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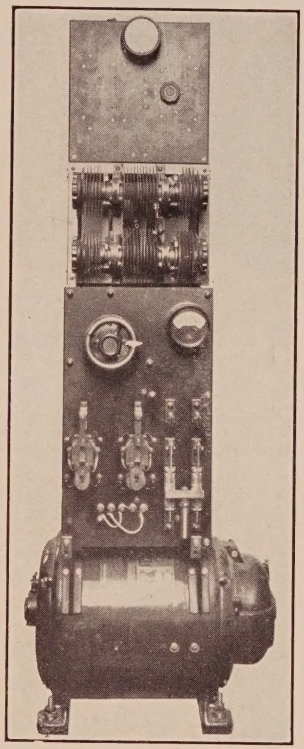

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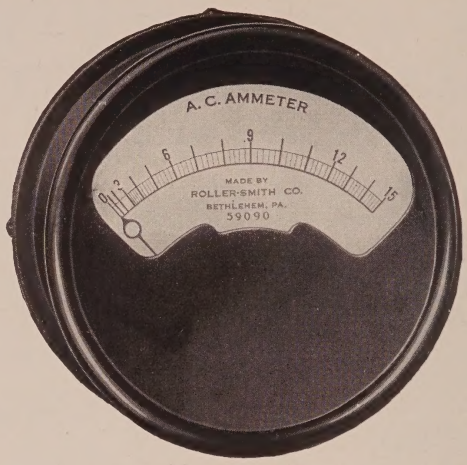


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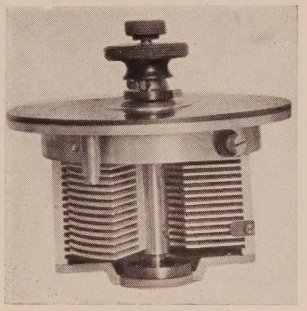

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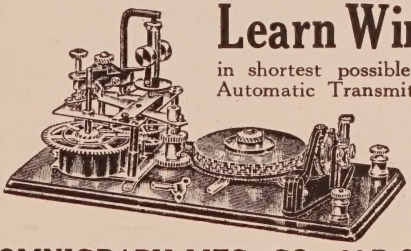

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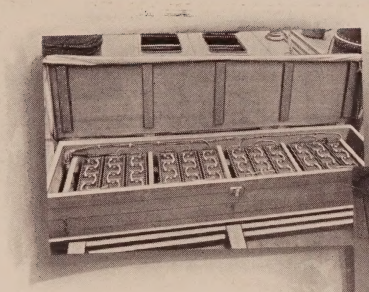

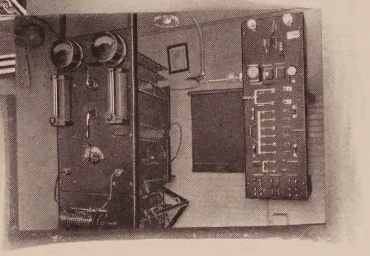

**Left:**—Exide Battery, Type MV 11 60 cells, on deck of S. S. *Comus* Southern Pacific Company.

**Below:**—Standard wireless switchboard and 2-KW. Marconi transmitter in wireless room of S. S. *Comus*.

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PROCEEDINGS OF
# The Institute of Radio Engineers

Volume 9 APRIL, 1921 Number 2

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## GENERAL INFORMATION

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# CENTRAL STATIONS FOR RADIO COMMUNICATION*

By

Ernst F. W. Alexanderson

(Chief Engineer, Radio Corporation of America; Consulting Engineer, General Electric Company)

Radio achievements are often referred to as belonging in the realm of mystery, and it is indeed wonderful that we are now able to speak with a voice that carries thru empty space across the oceans. Whenever knowledge conquers a new force of nature for the use of humanity, it ceases to be a mystery, but the pursuit of this knowledge makes an even greater appeal to the imagination.

The development of the steam engine was a triumph of the engineering art of the last century, but it was not the engine itself but the steamship and the locomotive that interested humanity.

The telephone and cables no less than the steam engine have introduced a new era in human affairs. They have, to a degree, conquered space and time, but only with certain serious limitations.

An ocean cable runs only from one landing place to another and it can be cut in time of war; its use can be censored by its owners and controlled by military and naval power. When, on the other hand, you send a radio message, it may reach all parts of the world. Depending upon whether it has been sent in code or in plain language, it may be a confidential, private message or a press message intended for the world at large, but nobody can directly prevent the electromagnetic waves themselves that carry the message from reaching their destination. It is thus not exaggeration to say that the emancipation of the human spirit that was begun by the invention of the printing press has found its fulfilment in radio communication. Radio makes the transmission of ideas from man to man and from nation to nation independent not only of any frail material

* Received by the Editor, October 12, 1920. Presented before a joint meeting of The Institute of Radio Engineers and The New York Electrical Society, New York, November 10, 1920.

carrier such as a wire, but above all it renders such communication independent of brute force that might be used to isolate one part of the world from another.

These are the ideal aims which inspire the engineers engaged in the development of the radio technique. This is also the explanation why some of the foremost lawyers, executives, financiers, officials, and statesmen of this country have found incentive in the human aspects of the radio technique to devote a great deal of their time and thought to its promotion and development on a world-wide scale.

The interest that is evidenced by all concerned in this subject has become much more serious since it has been established that the laws and forces with which we are dealing are within the control of our knowledge, so that engineers can now proceed with the design of a radio communication system with practically the same deliberate accuracy as in the design of an electric power transmission from a water fall to a railroad.

This audience is constituted of members of a society of electric power and light engineers as well as members of THE INSTITUTE OF RADIO ENGINEERS; so I shall take the opportunity to trace the close connection which now exists between electric power engineering and modern radio engineering and will demonstrate, as the specific subject of this paper, how the development of the Central Station for radio communication is as logical and inevitable as was the development of the central electric power station.

The entry of the Corporation with which I have been connected for the last twenty years upon the field of radio communication has been a gradual growth and a natural consequence of its general activities in power engineering. The engineers specializing in alternating current technique were in a natural position to take up the problem of designing alternators and transformers in the radio technique. These differ from the one used in the power technique principally in the fact that the number of alterations per second is about one thousand times as great. This speeding up of the performance one thousand times involved many new problems, but the most remarkable fact to record is that *the generally established principles of the alternating current power technique could be applied to the radio technique almost without change.* It meant that the magnetic properties of iron which had been reduced to an exact science by Steinmetz thirty years ago had to be studied again at radio frequencies; but it was found the Steinmetz

laws of hysteresis eddy currents, and skin effect were as accurate at two hundred thousand cycles per second as at twenty-five cycles.

It was furthermore found that the established conceptions of phase displacement, power factor, and leading and lagging currents were as applicable and useful in the high frequency as in the lower frequency technique.

It is true that radically different methods had to be devised for measuring power factors of a fraction of one per cent from the methods used for measuring power factor of 50 to 100 per cent, but the new methods of investigation verified the well-known principles.

The starting point of this development work was the time when Fessenden brought to the General Electric Company the problem of generating alternating currents for radio transmission. In doing so Fessenden realized that a practical solution of this problem could be worked out only by an organization of specialists.

Some of the problems that presented themselves in the evolution of the radio power plant were:

The design of a dynamo-electric machine or alternator generating electric power in the form of alternating currents of frequencies one thousand times as great as those used for motors and lights.

The development of magnetic amplifying devices capable of translating telephone and telegraph currents into corresponding modulations of the radio frequency energy flowing from the power plant into the radiating antenna.

The development of a regulator so sensitive as to hold the speed of an ordinary induction motor constant within a few hundredths of one per cent, this being necessary in order to maintain the proper phase relations in a load circuit working at one-third of one per cent power factor.

Improvement of the tuning of the antenna so as to transform as large a part as possible of the generated energy into electromagnetic waves.

The realization of Fessenden's vision, the radio power plant of to-day, became thus the result of the combined effort of leading electrical and mechanical engineers. Among these, it is sufficient to mention Mr. W. L. R. Emmet, the creator of the giant electric power stations of today.

The radio power plant which resulted from this was shown to Senator Marconi during a visit to Schenectady, and because of

his interest in its performance, it was transferred to the Marconi Radio Station at New Brunswick, where it had no sooner been installed than it was taken over for war service by the Navy. Further enlargements and developments of this installation which were undertaken by the Navy resulted in the plant which is now owned and operated by the Radio Corporation of America.

Here we had arrived at a point where two schools of engineering pursuing different aims, with widely different modes of thought, had been brought before a common problem. The one had been thinking in terms of power factor, kilowatts, and phase displacement, the other in terms of wave length, decrements, and tuning.

A third school of knowledge was at that time brought into contact with this technique and added new impetus to it. As soon as such scientists as Drs. Coolidge and Langmuir began to study the Fleming value and the remarkable little device invented by Dr. Lee De Forest and known as the audion, the foundation was laid for the vacuum tube technique which has so profoundly influenced the art of radio communication.

These scientists tell us that electricity is not the mysterious "power fluid" that we may have imagined flowing smoothly in our wires, but miniature plants or comets of condensed material electricity of definite charge and mass shooting across a miniature universe inside of a glass bulb and following orbits that can be calculated as accurately as the orbits of the stars.

Keeping in mind the origin of the modern art of radio communication in these three widely separate realms of knowledge, power engineering and electro-physics, we may now proceed to examine the essential parts. We find then—first, a modern electric power plant working at very high frequency; second, a network of wires a mile (1.6 km.) long, supported on tall masts; third, on the opposite side of the ocean a little glass bulb full of shooting stars. The question is: what does really happen?

Does the electricity generated by our alternator emanate from the antenna and flow in an undulating stream thru the air or thru the water or thru both? If we search for it in an airplane, we find it, and if we submerge ourselves in a submarine and search for it, we find it, and yet we are told it is not so.

Does the little electron, as an individual, take a leap off the aerial wires and, after devious paths, find its home in the glass bulb on the other side of the ocean? We are also told that it does not.

If I knew exactly what really does happen, and should try to tell you, then sooner or later somebody would claim that I was altogether mistaken. Therefore, I will only try to tell you how I imagine that it happens, wondering if any of you will see the same mental picture of the process that I see.

We were once told by the physicists that all space was filled by a fine substance that was called ether, and that the light and heat that radiated from the sun was a wave motion in the ether. The physicists now tell us that there is no ether, but still they say that light is a wave motion. Be this as it may, for the purpose of visualizing what takes place in radio transmission, it is convenient to cling to the theory of the ether.

We are familiar with other forms of wave motion—the air waves that carry sound to our ears and the water waves on the ocean. Thus the carrier of the radiated electric energy must not be likened to the flowing stream of water, or to the wind or to a bullet shot from a gun, but likened to a wave in a uniform medium where each particle of the medium oscillates around a stationary base line while the wave rolls forward.

The distance that a wave can travel in an absorbing medium before it fades out to a definite extent is proportional to its length. We may therefore introduce the idea of wave length, which is the distance from the crest of one wave to the next. The long swells of ocean travel for hundreds of miles, whereas a pebble dropped on a still surface of water produces a ripple that fades away in a short distance.

In radio communication it has been observed that the distance over which reliable communication can be maintained is about 500 times the length of the ether wave that is used. It may be more than a mere coincidence that the distance to which a sound wave travels in air, and a wave on the surface of water will travel before it fades out, is also about 500 wave lengths. The average wave length of sound of spoken words is abont one foot (0.3 m.), and we know that if we speak loudly our voices will carry a distance of about 500 feet (150 m.). The exceptions to this rule that will occur to anybody are also significant. We know what distances voices will carry over a lake in a quiet evening. We also know what extraordinary distances radio signals will carry sometimes in a quiet night. These are "except tions that prove the rule," and the rule refers only to reliable communication under normal conditions.

A radio transmitting system is designed for the purpose of producing waves in the ether which we call electromagnetic

waves, and for controlling the rate at which the waves are produced in such a way that a train of successive waves will carry the meaning of articulate speech or telegraphic code. If we wish to send a message a long distance, we must select a long wave. The distance to Europe is 5,000 kilometers (3,200 miles). If this distance is to be bridged by 500 wave lengths, each wave length must be at least 10 kilometers (six miles), or, as it is usually expressed, a wave length of 10,000 meters.

We can produce water waves by rocking a boat. If we rock a canoe rapidly we get a short wave, but if we rock a larger boat more slowly we get a correspondingly longer wave. To rock the boat requires energy, but in order to produce a wave of suitable length, the energy must act thru an intermediate member which has suitable size, proportions and period of oscillation.

In radio transmission the energy is furnished by the radio frequency power plant, but, in order to transform this energy into waves, there is required the intermediate member which makes contact with a large volume of the medium which carries the wave motion. This medium is the ether and corresponds to the water or the air in the more familiar forms of wave motion. The member that transforms the energy to the ether is the antenna. The waves used for trans-Atlantic communication are as a matter of fact 10,000 meters long, or even longer. The antenna corresponds to the hull of the rocking boat or the sounding board of the piano.

The analogy with water waves may be carried still further. The wave is a successive displacement of the medium, and the initial displacement produced by the member acting upon the medium is proportional to its volume. The water displacement of the boat corresponds to the effective volume of the antenna. The maximum voltage at which the antenna can be operated corresponds to the maximum angle to which the boat may be rocked before it ships water. This is the voltage at which the surrounding air breaks down under the electrostatic pressure. In electrical units, the displacement in the ether is expressed in meter-amperes. This is really a measure of volume as is apparent from the consideration that the amperes charging current at the limiting voltage is proportional to the two horizontal dimensions. The third dimension or the height appears directly in the product, and is expressed in meters.

The height of the antenna is the most expensive of the three dimensions by which we may create electric displacement in

the ether. The tendency in stations designed for greatest economy is, therefore, towards structures of moderate height and great length, whereas, the tendency in the past, when dynamic efficiency was the principal consideration, was towards towers of great height. The unit of performance on the old basis was kilowatts consumed by the antenna. The unit on the new basis is "ether displacement." This modern measure of antenna radiating capacity is the number of meter-amperes of "ether displacement" that can be produced at the voltage which is limited by the breakdown of the air.

The antennas of the stations of New Brunswick and Marion which are now used in trans-Atlantic service are each one mile (1.6 km.) long. In the new Radio Central Station, which is being built by the Radio Corporation on Long Island, there will be ten or twelve antennas, each a mile and a quarter (2 km.) long. This station is intended to communicate efficiently with all parts of the world. When very long distances are to be spanned, correspondingly long waves will be used. For efficient transmission of these long powerful waves, an antenna will be needed that makes contact with a large volume of ether. This will be accomplished by combining several of these antennas into one unit. At other times the same antennas will be used for the simultaneous transmission of several messages over shorter distances.

The shifting of radiation power which has been referred to is made possible by the use of the multiple tuned antenna which has been described in a previous paper before The Institute of Radio Engineers.[1] The New Brunswick and Marion antennas are now tuned so that each acts as six single antennas operating in multiple. The combining of several such groups in multiple is only a further extension of the same principle.

When two such antenna groups are connected in multiple, the loss resistance is reduced to one-half. Hence the efficiency of the antenna is increased so that a given power produces more radiation. Still more important is, however, the fact that more power may be utilized at this increased efficiency, and so the net result is that the amplitude of the radiated wave is doubled, which means that four times as much energy is radiated.

The economical factors that point to the radio central station as the practical solution of the problem of long distance communication are practically the same as those that created the central

[1] See Proceedings of The Institute of Radio Engineers, volume 8, number 2, pages 279–282.

electric power station. Broadly speaking, they provide for the utilization of the plant investment and operating force to the utmost by shifting the equipment from one service to another and combining it to meet various demands.

New York is a natural communication center and the service must extend to Europe, South America, and westward. Another Radio Central Station at Hawaii is being equipped to serve as a relay for all points on the other side of the Pacific Ocean.

While it is winter on the northern hemisphere, the radiating power to Europe can be much reduced, but this is the season when the South American traffic requires a maximum radiation because of summer conditions then existing on the southern hemisphere. The New York Radio Central Station can then divert some of its radiating power from the European to the South American circuits. There will also be daily fluctuations in traffic load which will occur at different hours due to the difference in geographic longitude. Thus the peak load of European traffic will occur at different times than the South American and Western traffic. The central station equipment can be utilized so as to take advantage of this.

The realization of trans-Atlantic radio telephony for commercial purposes is another object of the Radio Central installation. Trans-atlantic telephony will, no doubt, be something of a luxury for the immediate future. The radiation intensity needed for telephony is much greater than for telegraphy, and a plant designed purely for telephony might prove very expensive. However, the flexibility of the Radio Central, where any number of antennas can be combined when desired to produce a more efficient radiation, will make an extra powerful transmitter available when needed, while the plant may be used in a more economical way at other times for telegraphy.

SUMMARY: There are considered the mechanism of radiation and reception in radio communication. The design of the transmitting equipment is compared with the design of the usual alternators and power plants of electrical engineering. The main problems encountered are described, and an account is given of the solutions obtained. The development of the Radio Central Station for telegraphy and telephony is discussed, its arrangements described, and its usefulness indicated.

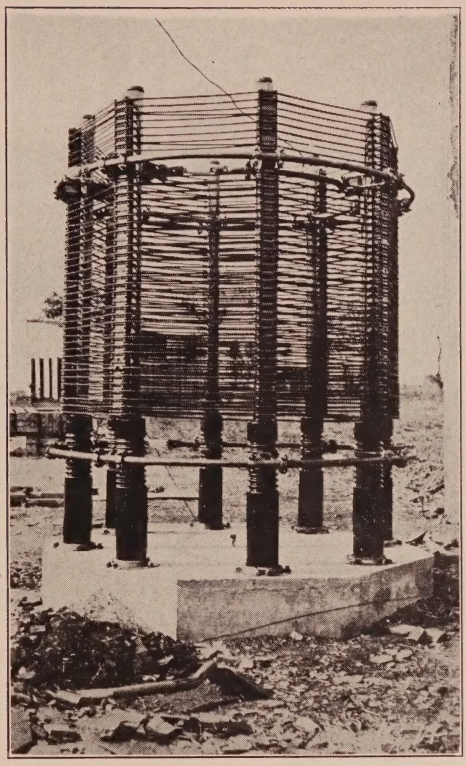

FIGURE 1—One of the Outdoor Loading Coils Used on the Multiple Tuned Antenna

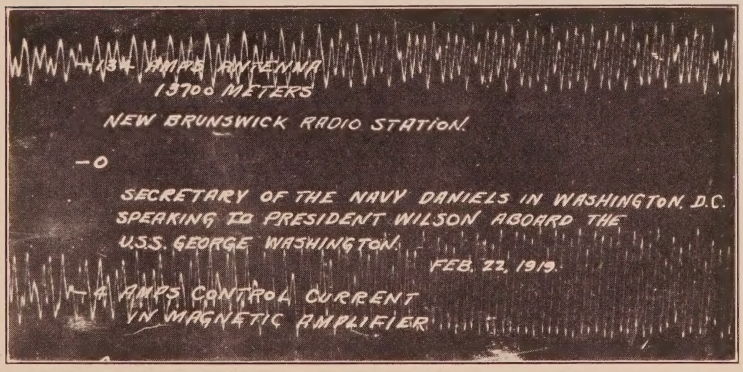


FIGURE 2—Secretary of the Navy Daniels in Washington, D.C., Speaking to President Wilson Aboard the U.S.S. *George Washington*

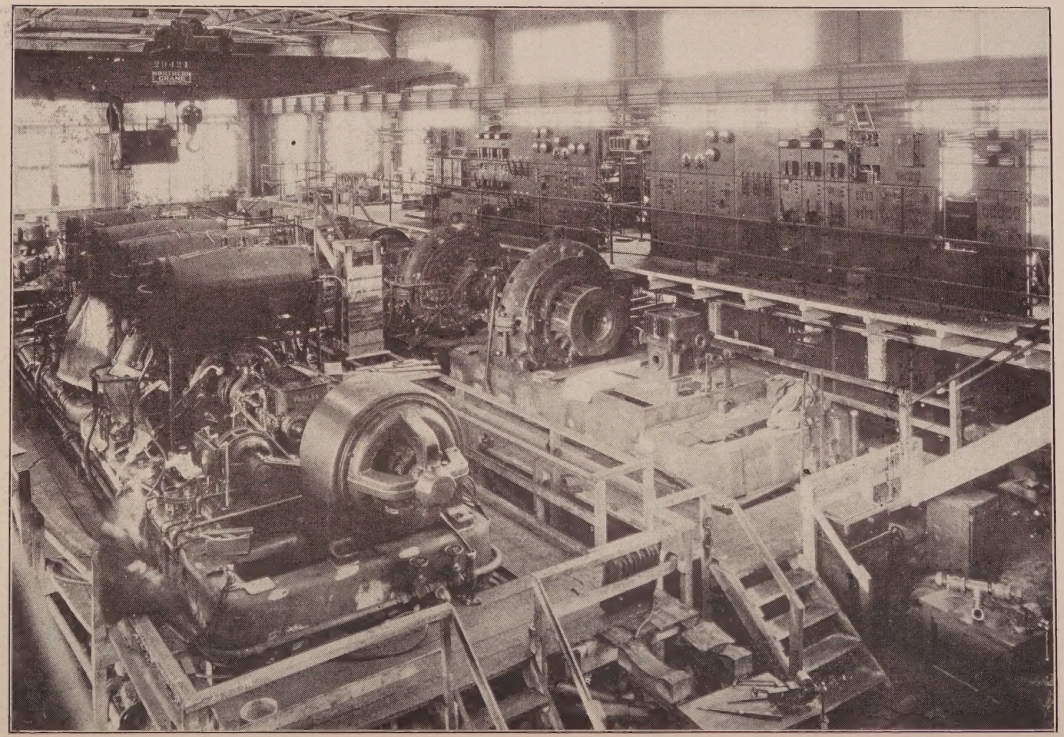

FIGURE 3—Section of the Building Where Radio Power Plants Are Being Manufactured at the Rate of Two Per Month

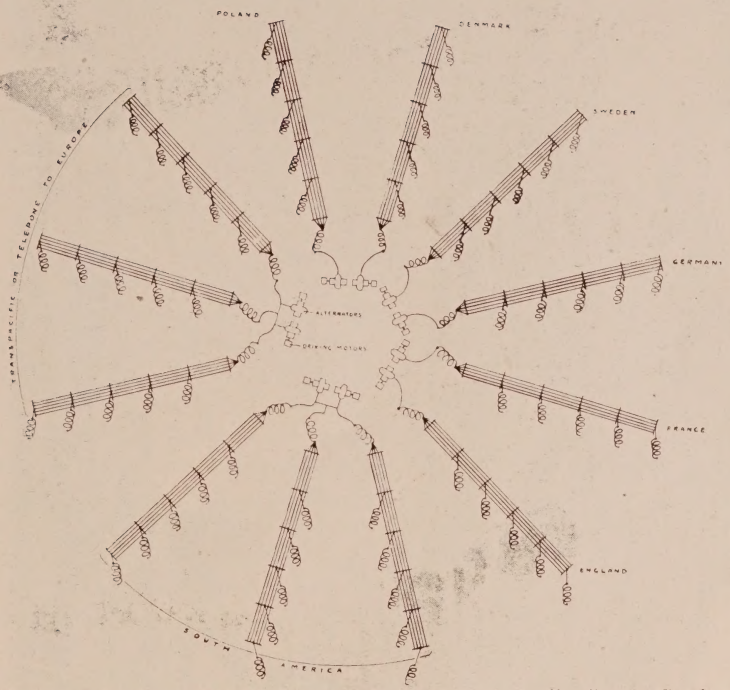


FIGURE 4—Antenna Combinations for the New York Radio Central Station

FIGURE 6—New York Radio Central Station

# THE SPECIFICATIONS AND CHARACTERISTICS OF MOORHEAD VACUUM VALVES*

By

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and

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The war has been responsible for some remarkable developments in radio apparatus, among which the vacuum valves have played an important part. The varied uses to which valves have been applied has necessitated the manufacture of large quantities of valves having uniformity of operating characteristics.

The British Government required a considerable number of vacuum valves, and while they were being built by hand in England, the supply was not sufficient. The valve adopted by the British Government was presumably copied from the French type, and complete specifications were submitted by the British Government which cover a large amount of detail. The writers will discuss part of these specifications in the following, calling attention to the careful mechanical measurements contained therein.

The Type "R," which is the British receiving tube, is of the three electrode type. The anode consists of a rectangular sheet of pure nickel 31 mm. $\times$ 15.2 mm. (1.22 $\times$ 0.6 inch), rolled into a cylinder 10 mm. (0.39 inch) external diameter, the thickness of the metal being 0.2 mm. (0.08 inch). The grid consists of a nickel wire 0.25 mm. (0.10 inch) diameter and 165 mm. (6.5 inches) long. This wire is bent into a spiral of 11 turns. The pitch of this spiral being 1.5 mm. (0.06 inch), the internal diameter is 4.25 mm. (0.17 inch). The filament is pure drawn tungsten

*Delivered before the San Francisco Section of The Institute of Radio Engineers, November 14, 1919. Received by the Editor, December 8, 1919.

wire, diameter 0.061 mm. (0.0024 inch) and 23 mm. (0.91 inch) in length when uncrimped.

The disposition of the electrodes is made as follows: The grid spiral and anode cylinder are placed so their axes are coincident with the filament. The grid is set so that 9 of its turns are within the cylinder and one turn projects at each end of the anode.

The size of the glass bulb specified is not to exceed 54.76 mm. (2.15 inches) external diameter, and the length from the remote end of the pins on the base to the tip 111.91 mm. (4.41 inches).

The British and French type of base is used, but was improved upon considerably by base manufacturers in this country. An unusual degree of mechanical accuracy was required in the base and terminal pins, a few figures for which are given here. The diameter of the terminal pin is 3.171 mm. (0.125 inch) and a tolerance of but 0.075 mm. (0.003 inch) is allowed. The length of the pin projecting from the base is 17.462 mm. (0.688 inch). Numerous test gauges are used in checking the assembled base and valve.

The electrical tests specified are characteristic of the mechanical details as contained in the specifications. To test the correctness of the disposition and proper dimensions of the elements, the filament is supplied with four volts, the anode being maintained at 80 volts positive with respect to the negative end of the filament, and a curve obtained by plotting as abscissas the potential difference of the grid in respect to the negative end of the filament and as ordinates the filament plate current. This curve must be a straight line for variations of grid potential between (minus three) and (plus twenty-five) volts, a variation of 1 volt grid potential producing a variation of plate current of at least 0.2 milliamperes. Furthermore, when the grid and plate are connected together and a potential of eighty volts applied to them, the current then obtained between the filament and these two electrodes must be above 8 milliamperes.

The bulb must be so evacuated that the "backlash" is between 0.5 and 0.02 microamperes. The method of obtaining the backlash is described here. A potential is applied to the grid, the anode being at 160 volts positive, and filament voltage 4. A curve is plotted, taking the grid current as ordinates and the negative grid voltage as abscissas. The negative current represented by the ordinate when the grid potential is 2 volts negative in respect to the filament, is the backlash. Further testing when the grid potential is removed, all other conditions the same, the

positive current to the grid must not exceed 1.5 microamperes.

The electrodes and all internal parts of the valve must be so freed from gases that no deterioration of the vacuum occurs when the anode is dissipating 15 watts energy for three minutes continuously. The anode dissipation is measured in the following way: The filament is supplied with 6 volts, the anode with 400 volts, and the grid with positive potential until the plate current reads 37.5 milliamperes. During the continuance of this test, no blue glow must appear in the bulb.

The filament current when six volts are applied must be 0.84 ampere with a tolerance of 0.035 ampere. The contact of the leading-in wires with the elements must be such that, when the valve is used in a 4-stage amplifier, no crackling sounds are heard. The insulation of the base must exceed 150 megohms when the valve is not lighted. The valve has a life of 800 hours when four volts are used on the filament.

The British Type "B," which is a transmitting valve, will next be considered.

The anode is a rectangular sheet of nickel 31 mm. (1.22 inch) by 16 mm. (0.63 inch), rolled into a cylinder of 100 mm. (0.394 inch) external diameter, the thickness of the metal being 0.22 mm. (0.0087 inch). The grid consists of molybdenum wire, diameter 0.2 mm. (0.008 inch) and length 330 mm. (1.3 inch). This wire is bent into a spiral of 22 turns, the pitch being 0.75 mm. (0.040 inch), the internal diameter being 4.1 mm. (0.16 inch). The filament is drawn tungsten containing a small percentage of thorium, the diameter of this wire being 0.058 mm. (0.0023 inch) and the length 22 mm. (0.87 inch).

The valve is assembled like the Type "R," and the following electrical tests applied: disposition of electrodes, and correctness of dimensions. The anode is maintained at 600 volts positive; six volts are applied to the filament; the grid is maintained at the same voltage as the negative end of the filament. Then the plate current must be 17 milliamperes plus or minus 6 milliamperes. The grid is next supplied with 10 volts positive, and the plate current must exceed by 5.5 milliamperes that current observed in the previous case.

The vacuum is tested as in the type R case except that the anode must dissipate 50 milliamperes at 600 volts for three minutes continuously. During this test the anode becomes white hot. No backlash test is made, but the filament emission is measured by connecting the grid and plate together and applying 80 volts with respect to the negative end of the filament, which

is supplied with 4 volts. The current thru the valve must then exceed 5 milliamperes.

The filament current when 6 volts are applied must be 0.85 ampere with a tolerance of 0.04 ampere.

The valves were constructed of materials produced in the United States, and the specifications followed carefully. When the completed valves were tested, they agreed with all the electrical measurements described above.

The type R valve, however, had its elements mounted vertically. This was done as the specifications did not cover the position of the elements, and no samples were available in this country at that time. Samples did arrive, however, before the type B was made, and the elements were changed to a horizontal position to conform to the French sample.

The United States Navy type, SE-1444, was the next valve constructed, using the helical grid and cylindrical plate.

A few mechanical changes will be noted, namely, that the grid and plate supporting wires were separated as much as possible, and made of heavier material. The capacity was decreased by these changes, while the internal impedance and amplification constants remained about the same as the British valve.

This type of construction is still used in the valves supplied to the Marconi Wireless Telegraph Company, and known as the "Marconi V. T." However, the flash exhaust is not used in the Marconi V. T., class 2. That is, a vacuum permitting gas action is used in this particular class of valve.

The tendency of the art at this time is to decrease the power consumption of vacuum valves for receiving purposes, and develop a valve which shall consume less than 1 watt, still retaining the high constants of the larger valve. The filaments will be so accurately mounted that no manual adjustment will be required for the proper temperature and emission. The valve will be made much smaller, and a type of base with very low capacity will be used. This will also greatly decrease the cost of the present valve.

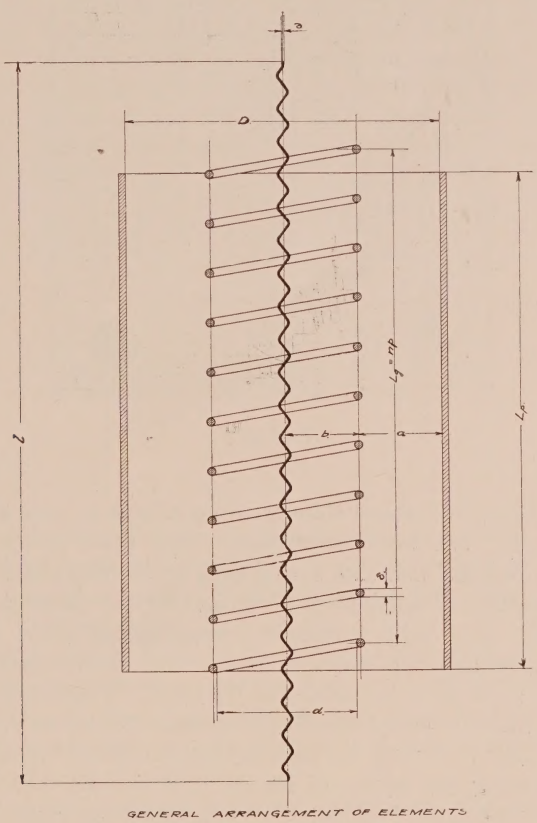
### The Characteristics of Moorhead Vacuum Tubes

The authors will not dwell on an historical review of vacuum tubes in general, nor will they go into a detailed description of tubes, such as the Fleming valve, kenotron, de Forest audion, pliotron, and so on because most radio engineers are familiar with these. For the same reason, and also because of detailed specifications given in the first part of this paper, further descrip-

tion of the Moorhead tube, or rather Moorhead tubes, will be omitted.

In general, the Moorhead tubes can be divided into two classes: the "soft" tubes and the "hard" tubes. To the first class belong the "Electron Relay" and "Class B," or "Class 1" tubes; to the second class belong the SE-1444, or "Class 2" tube, and the "Type C" tube, also the "British B" and "British R."

The "Key Figure" here given illustrates the general construction of these tubes and their dimensional nomenclature. The "Electron Relay" and the "Class 1" tube (also called "Moorhead Audion") belong to the class of detector tubes depending on the presence of traces of gas for their action. Figure 1 shows the representative characteristic curves of this type of valves.




KEY FIGURE—Construction of Moorhead Tubes and Principal Dimensions

Both the "Electron Relay" and the "Class 1" tubes are identical in their action, there being only a difference in mechanical construction.

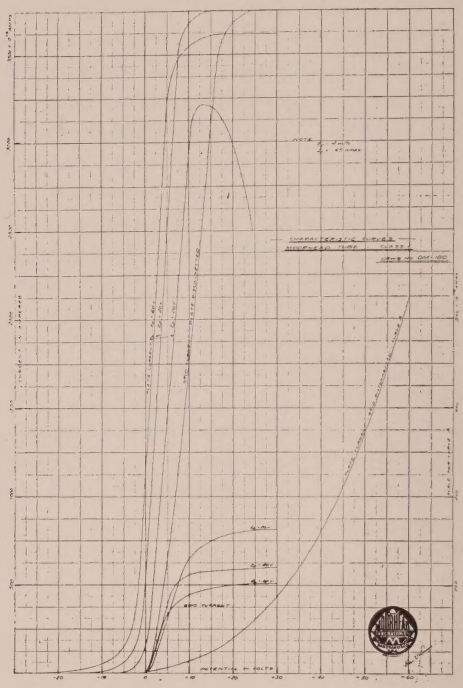


FIGURE 1

Paralleling the above class of tubes, a type of tube known as "British R" has been developed. This tube has a "hard" vacuum, but the elements are of such proportions as to permit the application of lower plate potentials for the use as a detector tube, this same tube making a comparatively good amplifier when higher plate potential is used. The characteristic curves of this type of tube are shown in Figure 2. These curves show that the plate potential should be between 20 and 40 volts when used as a detector with the grid potential about 2 volts.

Examining the curves of Figure 1, it becomes evident that the tube makes the best detector with about 20 volts plate potential and 0 volts grid potential; the higher plate potentials requiring higher negative potentials of the grid.

Comparing the curves of the two tubes we will find that the "Class 1" tube, or "Moorhead Audion," is a more sensitive detector than the "British R" tube.

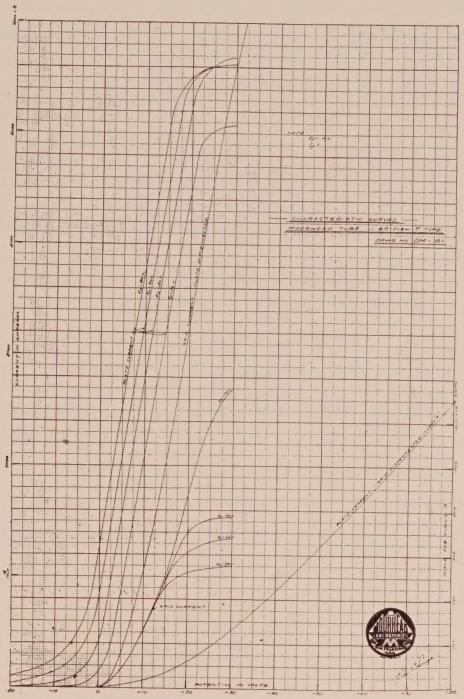


FIGURE 2

In general, as will be demonstrated later, two widely different designs of tubes should be adhered to: one for detector tubes and one for amplifier tubes. While the same tube may serve as detector and amplifier (for example, "British R" tube and SE-1444 tube), yet this is accomplished at a sacrifice of the best operating characteristics. From a number of tubes specially built for the purposes of research, the fact that a tube making the best amplifier makes a poor detector and conversely, has been brought out very clearly.

The authors' research work was undertaken chiefly with the purpose of developing a tube having high amplification and low resistance,—characteristics very much desired. The extent

of research work is shown by the accompanying plots of tests of a few of the tubes selected in such a manner as to show the extreme results obtained. The static method in all cases was used, which, altho erratic to some extent, permits calculation of coefficients, or constants, as defined in Mr. Ballantine's paper (Proceedings of The Institute of Radio Engineers, April, 1919).

Particular attention was paid to the determination of following constants:

$$\mu = \frac{dE_p}{dE_g} = \frac{\text{change of plate potential}}{\text{change of grid potential}} = \text{amplification constant.}$$

(The author suggests the term "grid control coefficient.")

$$R_o = \frac{dE_p}{dI_p} = \frac{\text{change of plate potential}}{\text{change of plate current}} = \text{internal resistance.}$$

$$\rho = \frac{\mu}{R_o} = \frac{dI_p}{dE_g} = \frac{\text{change of plate current}}{\text{change of grid potential}}.$$

All of the three ratios, which are called constants, are by no means constant, their values varying within wide limits for the same tube. The accompanying curves of Figure 3 give the magnitudes of such variations in values of an SE-1444 tube. These variations are by no means accidental, but are inherent characteristics of every tube. Particular attention may be called to

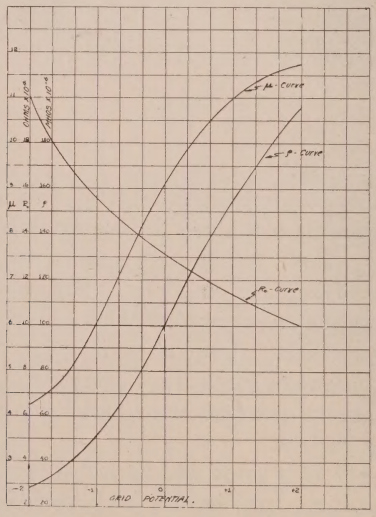



Figure 3

behavior of the two of the "constants," the amplification constant and the internal resistance. Every small gain in amplification results in an increase of internal resistance, and it does not seem possible to build a high power amplifying tube of low resistance.

Moreover, the same types of tubes, made in exactly the same way, with the same precautions during the process of manufacture, of the same materials, in short, tubes supposed to be identical in every respect, show variations with regard to the operating characteristics, in addition to the variations mentioned above. The magnitude of these variations is clearly shown in Figure 4.

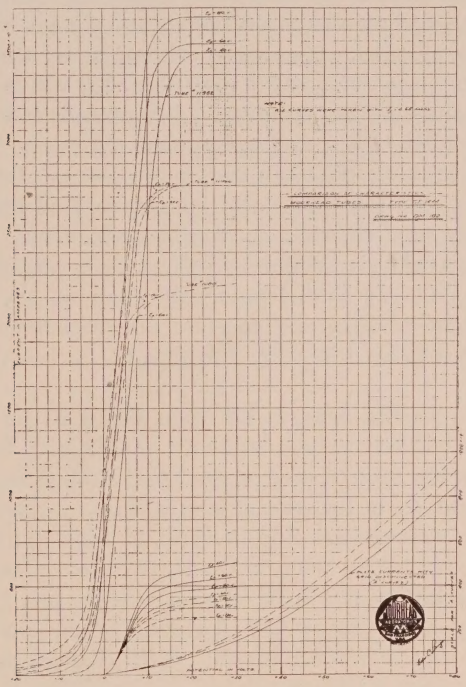


FIGURE 4

The three sets of characteristic curves plotted to the same scale very vividly show how minute defects, detection of which under ordinary conditions and with less sensitive apparatus would be termed "hair splitting," will change the characteristics of a tube.

The plate material strained a little while being rolled into the cylindrical shape and slightly opening when subjected to high temperature, will get away a fraction of a millimeter from the grid; the result is that the characteristic of the tube can hardly be recognized; slight variations of tension on the filament wire, uncertainty of supports retaining their shape when brought up to a temperature of about 800 degrees Fahrenheit (426° C.) during the exhaustion, or to a still higher temperature during the "blue-glow" process,—all these causes will have their effect. In addition, there are gases in the metal of tube elements, which gases, tho driven out of metal at very high temperature, defy the action of the "getter" and are re-absorbed by the metal to be driven out again when the tube is allowed to reach a high enough temperature—and we all know what a trace of a gas will do to the characteristic of tube. The result is that, to be exact, there is one chance in a thousand to get two tubes that will be absolutely alike. But this *is* "hair splitting." Thousands and thousands of tubes have been built and have given absolutely satisfactory service by slight variations of minor accessories of the circuits using a tube. On the whole, and for all practical purposes, the hard vacuum tube shows a consistent uniformity. A tube of definite design will give definite results within very close limits.

At this point, we shall temporarily abandon the discussion of general characteristics and consider the characteristics shown by each of the tubes tested by the authors, including even the soft tubes.

Up to the present date, as has been stated by an authority, quoted literally, "the vacuum tube art is in a very fluid state." The action of the tube in general has been studied and the tube has been made use of for several very different applications, all, more or less, successful. But in spite of wide applications of the tube, in spite of its being in use daily all over the world, no satisfactory solution of a mathematical theory has been given. The fact that all of the tubes (speaking of the three element tubes) have similar characteristics, their characteristic curves showing that they all belong to the same family, leads to the belief that one and the same law of action governs them all. Mathematical formulas have been advanced, but these formulas as tested by the authors, did not give satisfactory results. It is only to be hoped that more research work and more detailed study will be done in connection with the tube to arrive at the law determining the action of the tube, no matter what its shape, and no matter what its proportions may be.

In what follows, the authors will not try to advance any mathematical theory, but will confine themselves to presenting a few observations of the action of the tube elements, which actions being absolutely uniform may lead to a founding of a definite theory of the vacuum tube.

The first element to be mentioned is the tungsten filament. Taking measurements on different sizes of filament wire, both straight and crimped, the authors have found that the specific resistance of material of the filament was 0.000,005,58 ohms, or $5.58 \times 10^{-6}$ ohms per cm.$^3$, which result corresponds very closely with that given by the Bureau of Standards, which is given as $5.6 \times 10^{-6}$ ohms. The temperature coefficient was taken from the same source and used as an average of 0.0045 per degree Centigrade. It was found that the crimping added 27 to 30 per cent to the resistance, thus increasing the length of the filament wire about 30 per cent. Using a specific resistance $\rho = 5.6 \times 10^{-6}$ ohms, the curve of resistance of tungsten wire of different diameters has been calculated and is given in Figure 5.

All of the tubes used in the tests, except the "British B" tube, have 0.061 mm. (0.0024 inch) diameter filament wire 23 mm. (0.905 inch) between the supports. The average filament resistance is 0.565 ohm at about 20 degrees Centigrade; at this

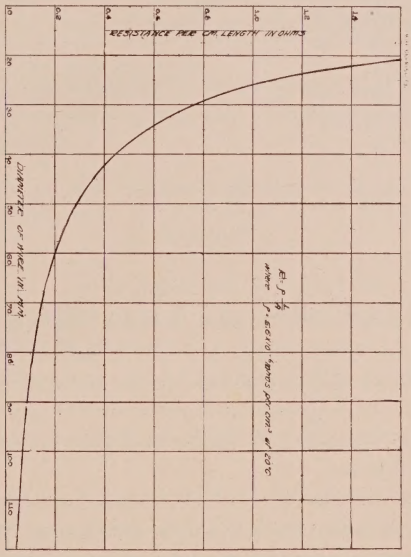



FIGURE 5

temperature the combined resistance of copper wire connections, platinum seals, and nickel supports measured 0.08 ohm. Figure 6 gives resistance, temperature and current curves of 0.061 mm. (0.00241 inch) and 0.058 mm. (0.00228 inch) diameter filaments. The temperature and resistance curves are those of 23 mm. (0.905 inch) long, 0.061 mm. (0.00241 inch) diameter filament. For practically all of the tests the filament voltage was kept at 4 volts.

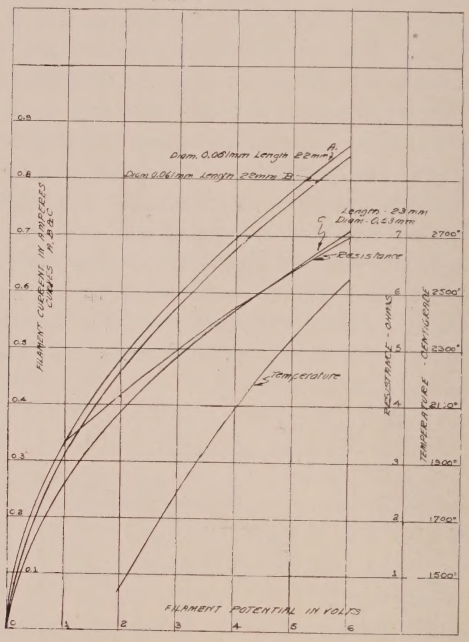



FIGURE 6

From Figure 6 it will be seen that the filament was at about 2, 100 degrees C. when using 4 volts, and that its temperature is raised to about 2,600 degrees C. using 6 volts. It was the authors' intention to give an account of every test at 4, 5, and 6 volts on the filament, but such comparative tests will be reserved for the future.

The electronic mission has been carefully studied by Richardson, who expressed the law governing the emission of electrons in an equation analogous to the equation of evaporation

of liquids. This equation appears below, together with sample calculations of the current. The two constants entering into the equation were taken as calculated for tungsten by Langmuir.

The length of the filament being 23 mm. (0.905 inch) between the supports, or about $23 \times 1.3 = 29.9$ mm. (1.18 inch) effective, we have for the exposed area of filament

$$A = \pi d l = 3.146 \times 0.0061 \times 2.99 = 0.057 \text{ cm.}^2 \text{ (0.00086 sq. in.)}$$

Figure 7 shows a graph of the "thermionic" equation given by Richardson which is

$$i = A T^{\frac{1}{2}} \varepsilon^{-\frac{b}{T}}, \qquad (1)$$

where $T$ = temperature of filament (Kelvin or absolute)

$$a = 23.6 \times 10^9$$

$$b = 52.5 \times 10^3$$

or

$$i = 23.6 \times 10^9 \, T^{\frac{1}{2}} \varepsilon^{-\frac{52500}{T}} \text{ milliamperes/cm.}^2$$

Calculated emission at 2,100 degrees C.,

$$i = 14.95 \times 10^{-3} = 853 \times 10^{-6} \text{ amps.}$$

The above results represent the maximum electronic emission available at the surface of the filament as the equation is strictly "thermionic" and does not take into consideration the presence of a charged body or the separating space between same.

On the other hand, we have an equation giving the maximum current in the cylinder with the wire filament at the axis of the same.

$$i_{max} = 14.65 \times 10^{-6} \frac{V^{\frac{3}{2}}}{r} \text{ amps./cm. length of the cylinder} \qquad (2)$$

$V$ = applied potential

$r$ = radius of the cylinder.

Calculating the current which would flow in a tube having 10 mm. (0.394 inch) diameter plate and 15.2 mm. (0.6 inch) long, we would have for 10 volts potential of the plate

$$i_{max} = 14.65 \times 10^{-6} \times 1.52 \times \frac{\sqrt{1000}}{0.5} = 14.1 \, (10)^{-4} \text{ amps.}$$

This equation does not take into account the temperature of the filament, which is not quite correct, as the temperature of the filament has an effect on electronic emission,

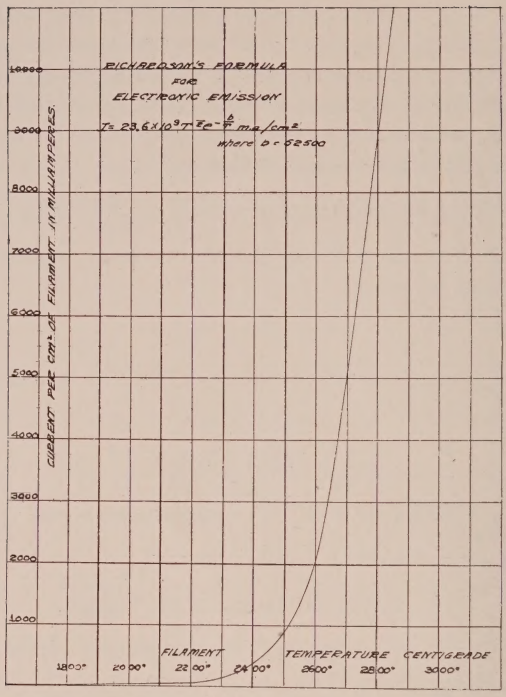



FIGURE 7

more noticeable as the potential increases. Figure 8 shows the plate potential curves of two hard vacuum tubes without grids at 4 and 5 volts filament potential; two soft tubes without grids were also tested and it will be noted that, up to the potential of 10 volts on the plate, the curves show uniform rise but for the higher potentials on the plate the saturation of the soft tubes becomes very marked. The difference between the 4- and 5-volt curves is very slight, demonstrating that the increase of the temperature of the filament does not materially increase the flow of the current. Carrying out the calculation for 20 volts plate potential, we will have

$$i_{max} = 14.65 \times 10^{-6} \times 1.52 \times \frac{\sqrt{8000}}{0.5} = 39.9\,(10)^{-4} \text{ amps.}$$

The theoretical curves have been plotted in Figure 8. The curve in dashes indicating the position the curve for 5 volts should be corrected as the voltmeter was later found to be reading 1.5 volts too high.

With the above correction, the theoretical and the actual curves are brought close together.

Examining the equation for the current of the tube with cylindrical elements, or

$$i = \frac{2}{9}\left(\frac{2e}{m}\right)^{\frac{1}{2}} \frac{V^{\frac{3}{2}}}{r},$$

we note that the diameter of the filament wire is not taken into consideration at all. The formula makes it appear that the cur-

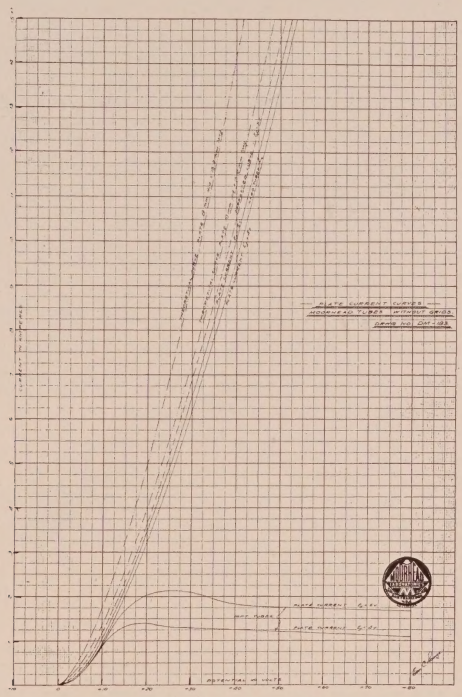


FIGURE 8

rent is altogether independent of the diameter of the filament, which, in the opinion of the authors, is not correct. As will be demonstrated later, the size, or diameter, of the filament is a very important factor in the analysis of the action of the tube and in determining the velocity of emission of the elementary charge. Reference may be had to the curve of forces due to an electric

charge on a cylindrical conductor such as filament. The formula gives higher results than are obtainable in actual tubes.

Figure 9 gives a series of plate and grid currents, with all elements assembled in place, except one which shows the curve of the plate current of a two-element tube (the same as Figure 8).

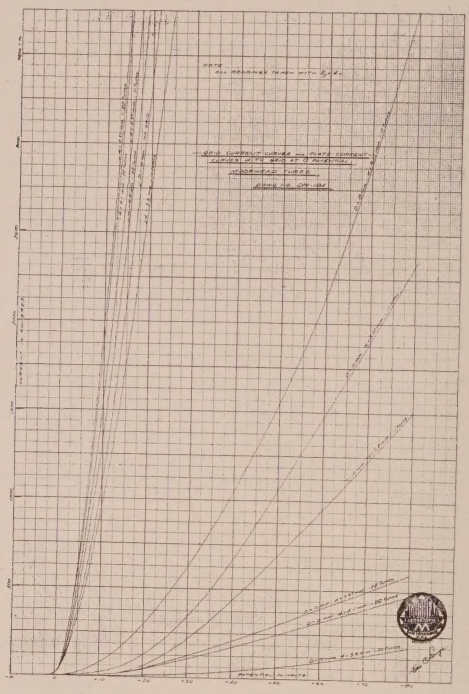


FIGURE 9

Studying the curves we will find that the plate-current curve of the gridless tubes runs together with grid current curves. All other curves represent the plots of readings of plate-filament current, with grid kept at zero potential by grounding it; when taking grid-filament current readings the plate was grounded. (No difference in readings was noted when grid or plate were not grounded). Each curve on this drawing very closely follows the equation

$$Y = A X^{\frac{3}{2}}$$

This equation appears to express the law governing the opera-

tion of any two-element valve, within the limits of saturation, The saturation point of the valve depends on the filament temperature and the potential difference between the filament and the anode. The difference between the results as given by the "thermionic" equation (1), and equation (2), is evidently due to acceleration of emission due to applied potential. This may also explain the difference between the slopes of the calculated curves and those plotted from actual tests.

It will be noted that checking the curves from point to point, the rise of the curves is not exactly in the ratio of $\frac{V_2^{\frac{3}{2}}}{V_1^{\frac{3}{2}}}$, but is more correctly in ratio of $\frac{V_2^{\frac{3}{2}}}{V_1^{\frac{3}{2}}} k$, where $k < 1$, and may be taken as a factor giving the difference of total potential applied minus the potential required to force the emission of electrons in excess of natural emission, or emission which would take place if the filament were removed from the influence of a charged body.

Figure 10 gives the plots of tests of the grid-filament current characteristics. The sketches of Figures 11 to 14 inclusive show the same curves to the smaller scale. From the curves, it is evident that the same law of the flow of current applies in this case also. Three different sizes of grids were tested, in each case the plate was grounded and thus kept at zero potential.

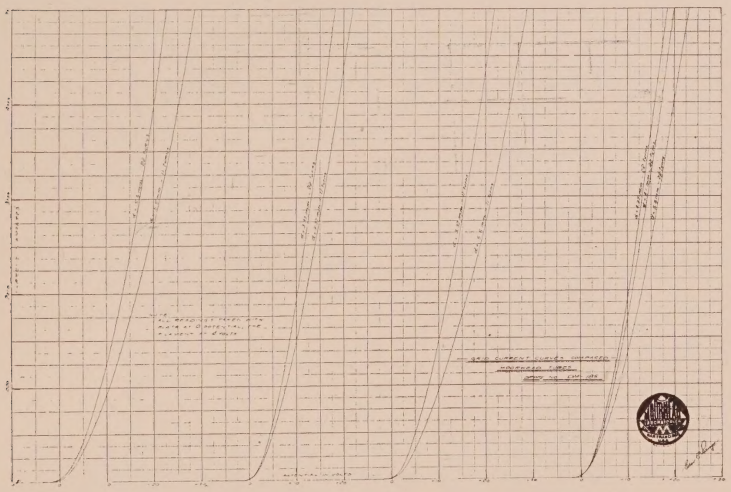
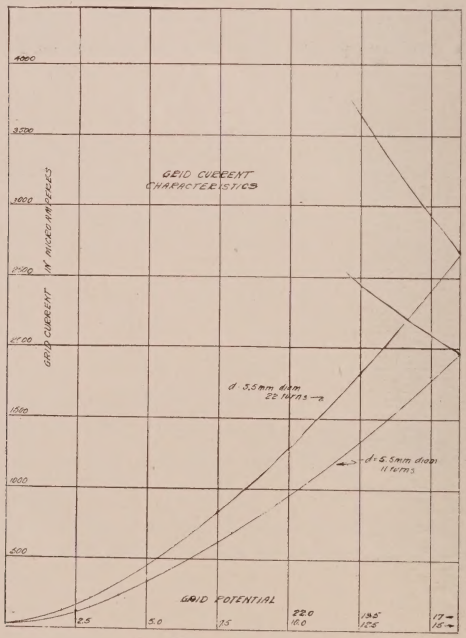


FIGURE 10

Figures 11 thru 14 represent the grid potential-grid current curves. The tests were taken with grids 11 and 22 complete turns, 3.37, 4.1, and 5.5 mm. (0.133, 0.162, and 0.217 inch) diameters respectively (these diameters are the diameter of mandrels used in the winding of the grids and, therefore, represent the inside diameter of the grid spiral). The wire used was $0.010'' = 0.254$ mm. diameter The pitch of winding for the 11-turn grid was 1.5 mm. (0.059 inch), that for 22-turn grid was 0.75 mm. (0.0295 inch).

Figure 15 gives grid potential-grid current characteristic of a tube having grid and filament only. The same drawing shows three computed curves for plates of length equal to the length of grids and of the same internal diameter. It will be noted that the ratio of values of plate current to that of the grid of the same diameter and length, depends on the number of turns of the grid and its distance from the filament.

Figures 11 and 12 (Curves 1 and 2 in Figures 13) show two sets of grid current curves; Figure 7 shows curves for 5.5 mm. (0.217 inch) inside diameter grid, Figure 11 for 3.37 mm. (0.133 inch)




FIGURES 11

diameter grid. These two sets of curves give the variation of current as a function of number of turns, and show that by doubling the number of turns of the grid we do not double the current. The difference between the values of current for the same size of grid but different number of turns appears to be not only a function

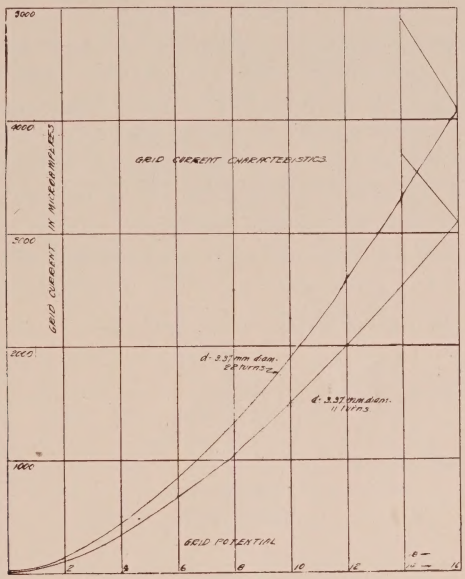



FIGURE 12

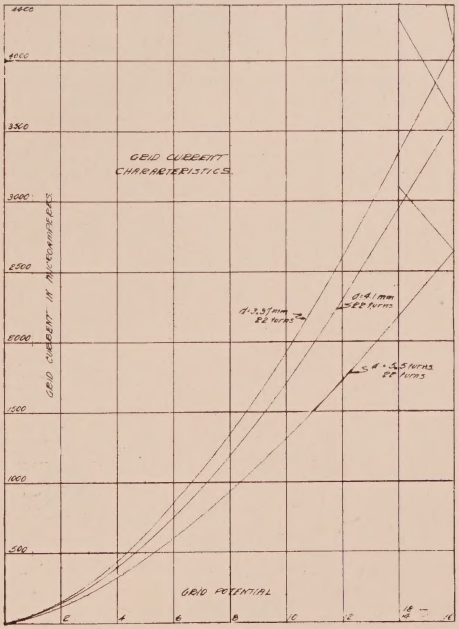



FIGURE 13

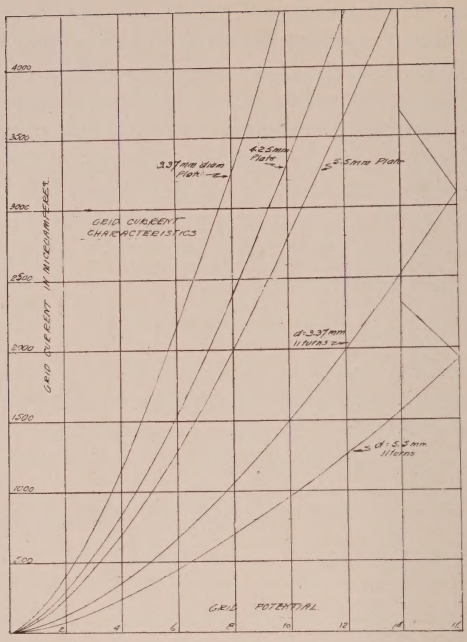



FIGURE 14

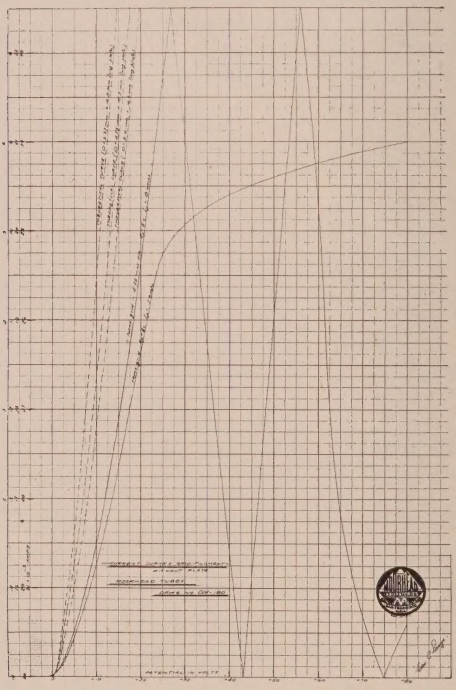


FIGURE 15

of the number of turns but also a function of its distance from the filament and the potential applied. The determination of this function, we believe, will completely solve the problem of mathematical predetermination of characteristics. Two curves in Figure 16 show the variation of current difference with varied potential for 11- and 22- turn grids, and it will be noted that the slope of the curve 3.37 mm. (0.133 inch) grid is considerably greater than that of 5.5 mm. (0.217 inch) grids. The variation of grid current as a function of the diameter turns is clearly shown in Figure 13, representing the current curves for 3.37, 4.1, and 5.5. mm. (0.133, 0.162, and 0.217 inch) diameter grids. The curves show a certain consistency, but do not follow exactly the ratios of their diameters.

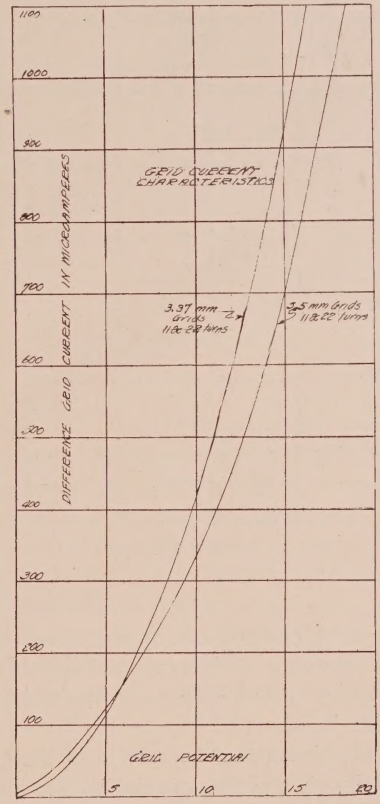



FIGURE 16

Figure 17 gives the curves of grid current differences as function of diameters of the grid and the potential applied. Here the curve for 22-turn grid shows a greater slope than that of 11-turn grid. It is impossible for the authors to present at present

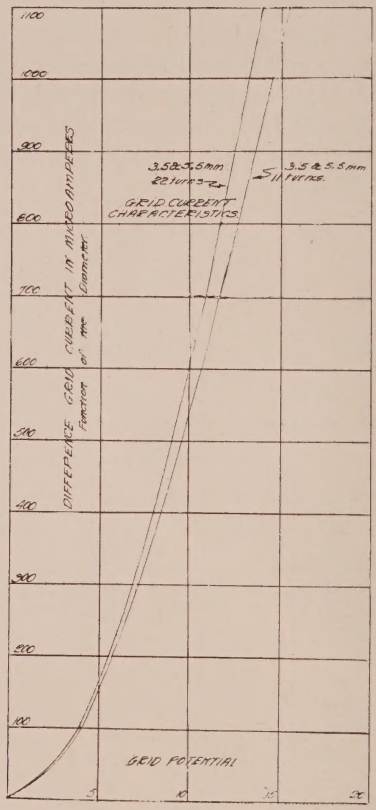



FIGURE 17

a complete analysis of grid action, as the observations and experiments are not complete. Several special tubes will have to be made and carefully tested before conclusive interpretation of the grid action may be given.

It is the writers' opinion that the action of the grid is dependent upon the distribution of the charge on the grid; due to its non-uniform distribution, the path taken by the elementary

charge diverges from a straight line, as it would if no charged body were present near the filament. The grid consisting actually of a number of rings will produce a field as indicated on the attached force diagrams, Figures, 18, 19, and 20. These force

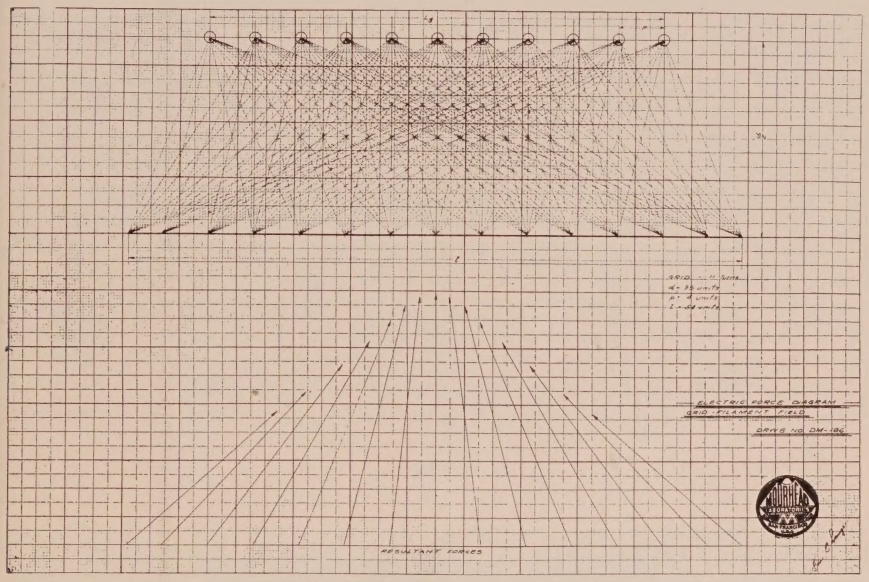



FIGURE 18

diagrams indicate that the moving charge, purely under the action of grid, without the action of any body placed outside the grid, will be crowded toward the center turns of the grid, this action becoming more and more apparent as the diameter of the grid decreases. The diagrams were drawn for arbitrarily assumed values for the diameter, spacing, and so on, with the sole intention of studying the resultant action of a number of rings, or turns, placed concentrically with regard to the filament. The curves on Figure 21 represent the values of forces from point to point extending thru the whole length of the filament; the values of the resultant forces were taken disregarding their direction, for 1, 3, 5, and so on turns of the grid. The curves are self-explanatory and show very clearly that with increasing number of turns and increasing diameter of the grid, its action on the filament (or, rather, its action in the space between fila-

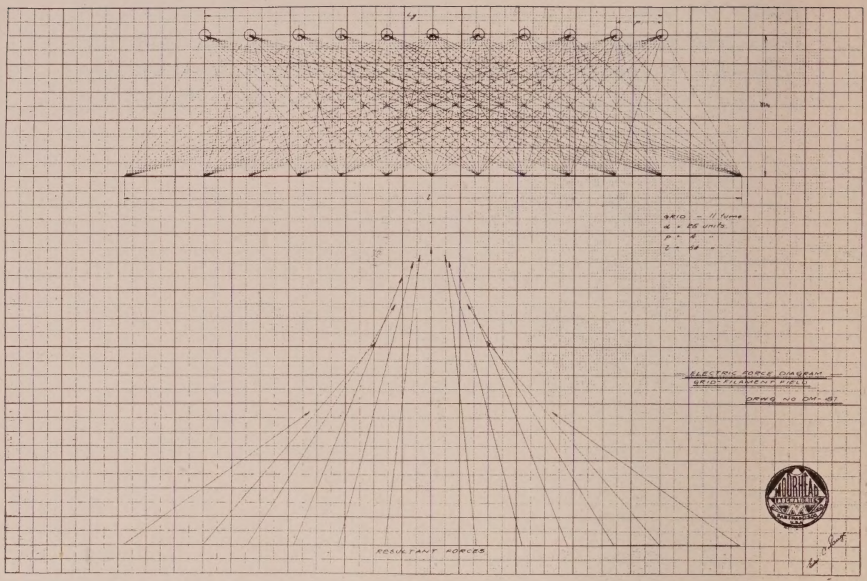



FIGURE 19

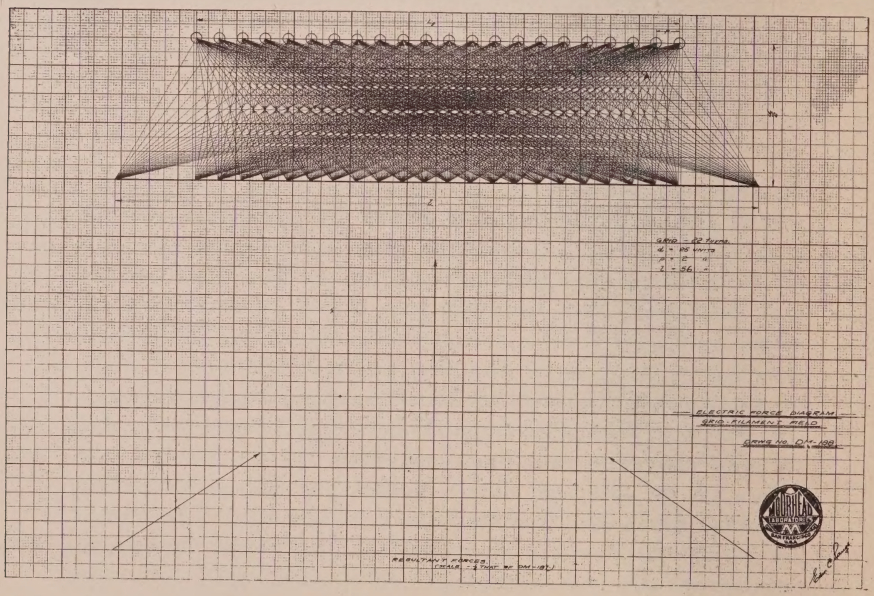



FIGURE 20

ment and grid) becomes more and more uniform. In general, the number of turns, their spacing, and the diameter of the grid and also length of the filament and its diameter, all have to be considered in an analysis of grid characteristics.

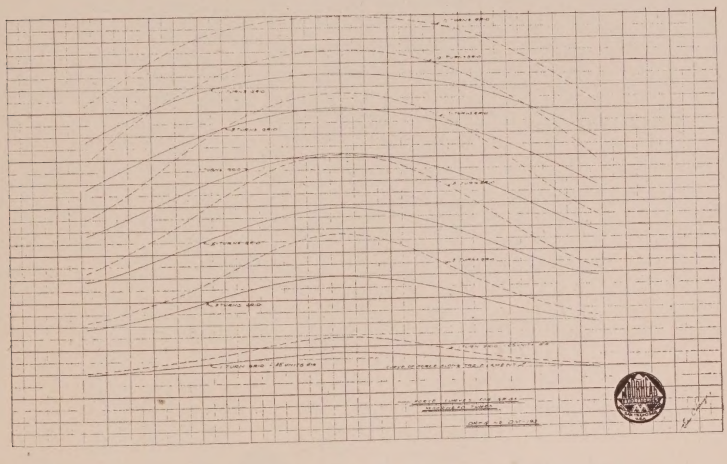


FIGURE 21

The equation (2), according to the above statement, can not be applied in the case of grid. Studying the characteristics of two-element tubes consisting of the filament and plate only (so-called gridless tubes,) it was observed that in each case the value of current as given by the formula was too high. This fact is very clearly shown by the curves of Figures 8 and 22, and Figure 8 gives comparisons of the plate-filament current of a hard vacuum tube with that of a soft tube at different temperatures of the filament, and also calculated curves for 8 and 10 mm. (0.315 and 0.394 inch) diameter plates. Figure 22 shows tests of 3 tubes with plates 8 mm. (0.315 inch) diameter. (It must be noted that the specified diameter of 8 mm. is not correct by a wide margin for these tubes; the plates were formed by means of a pair of pliers and then very poorly. The curves are shown solely for the purpose of demonstrating how small mechanical defects will alter the characteristics of a tube). All of the three tubes were exhausted at the same time and with the same precaution, yet they show considerable variation as regards the limits of saturation, indicating some condition practically beyond control.

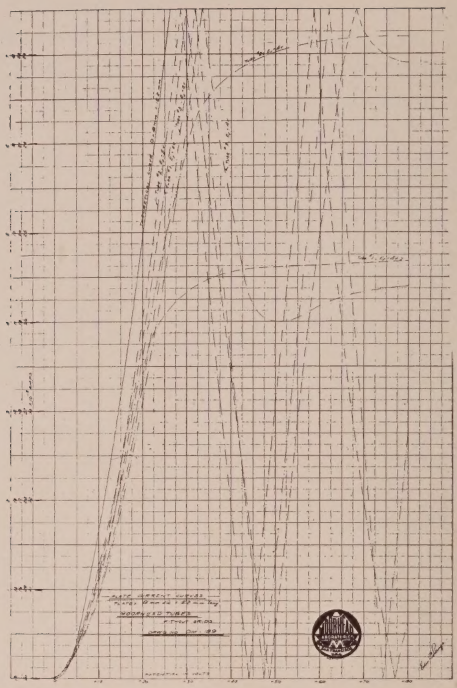


FIGURE 22

In general, it may be said that the plate-filament current at a given temperature of the filament in a two-element tube, consisting of a cylindrical plate and a filament, is a function of the plate diameter, and the diameter of the filament and its length.

Figures 23 thru 27 give the curves representing the values of plate-filament current of hard vacuum tubes, all tubes having the three elements completely assembled in accordance with standard practice. The readings were taken with the grid entirely disconnected (grounding of the grid did not affect the readings at all). Thus the action of the grid was confined to a purely screening action and the results obtained were rather startling. The tubes tested had diameters of the plates varying from 8 mm. to 10 mm., (0.315 to 0.395 inch) and three sizes of grids were used, 3.37 mm., 4.1, and 5.5 mm. (0.132, 0.162, and 0.217 inch) and varying the number of turns from 11 to 22. Due to lack of time, no valves were tested with grid turns between 11 and 22, but it is expected to complete the tests before

final mathematical analysis is completed. The determination of exact relations between the geometrical dimensions of the grid and its action on the plate is more difficult than may appear at first, but, in the opinion of the authors, a few more tests will definitely indicate the laws governing said action. Figure 9

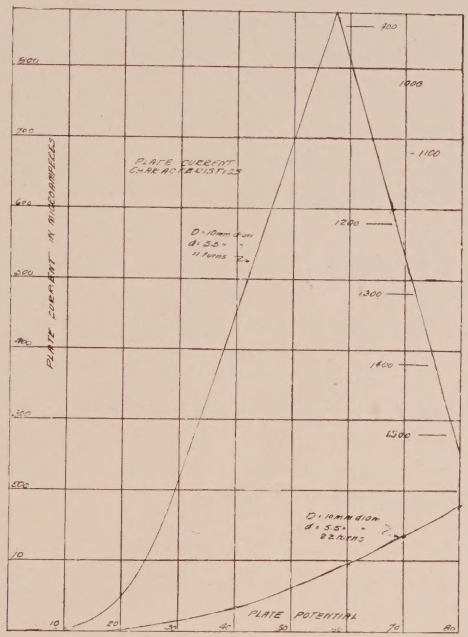



FIGURE 23

shows the plate-filament and the grid-filament current curves; the plate-filament curves were taken with grid disconnected, and are the same as shown on sketches, Figures 23 thru 27, and are plotted here to the same scale in order to bring out more clearly the action of the grid as a screen; the grid-filament current curves were plotted on the same sheet to demonstrate that one and the same law governs the flow of the current for both plate and grid. It is of interest to note that the plate-filament curve of a gridless tube (D = 10 mm. or 0.394 inch) takes its place in this family of curves among the current curves for grids.

A careful study of the curves will reveal several interesting points. Examining the grid-filament current curves, we note

that for the same diameter and the same overall length of grid, the current will be greater the greater the number of turns. The difference in current between two grids of same diameter but of different number of turns seem to be a function of the number of turns only, and is independent of the diameter of the grid. As expected, the larger the diameter of the grid, the smaller the current.

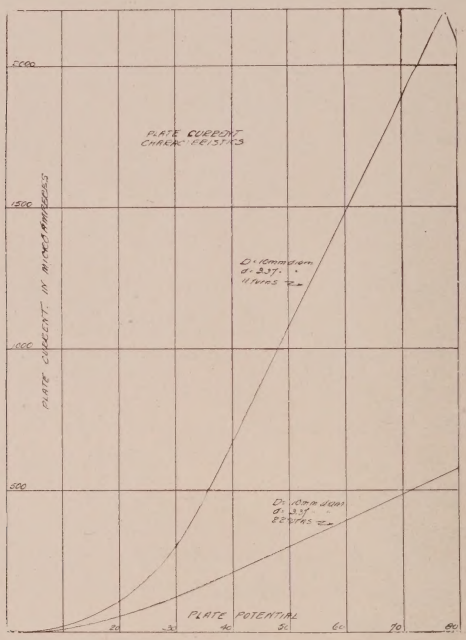



FIGURE 24

We will now compare the plate-filament current curves screened by the grid (grid at zero potential) in greater detail and refer to Figures 23 thru 27. Figure 23 represents current curves of a tube having 10 mm. (0.394 inch) diameter by 15.2 mm. (0.6 inch) long plate (D = 10 mm). The diameter (inside diameter) of the grid is $d = 5.5$ mm. (0.217 inch), the pitch of winding for 11-turns grid being 1.5 mm. (0.059 inch), and that for 22-turns grid 0.75 mm. (0.029 inch). These curves show that a slight difference in values at lower potentials of the plate increases very rapidly as the potential increases, the difference

varying approximately as the cube of the potential. The same approximate relation is brought out by the curves of Figure 24, which curves were compiled from readings of tubes of same dimensions as those of Figure 17 except that the diameter of the grid was made 3.37 mm. (0.133 inch). Both of the above sets of curves show the plate-filament current, with grid interposed, but at zero potential, as a function of the number of turns in the grid, when overall dimension of the grid and its relative position with respect to other elements were kept the same.

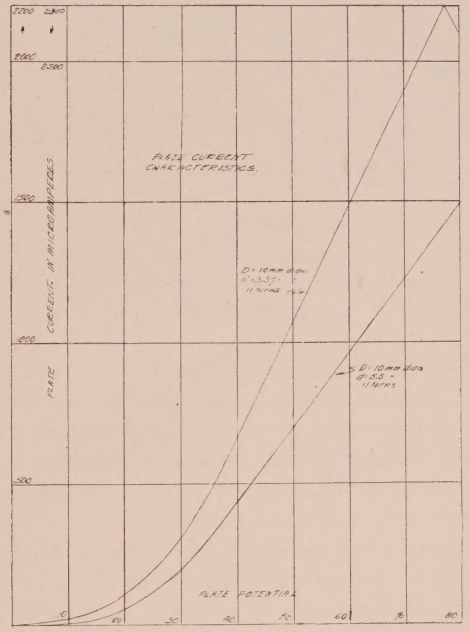


PLATE CURRENT CHARACTERISTICS
PLATE CURRENT IN MICROAMPERES
PLATE POTENTIAL
D = 10mm dia. d = 3.37 mm 11 turns
D = 10mm dia d = 5.5 mm 11 turns

FIGURE 25

To compare now the difference of the flow of plate-filament current, keeping the number of turns of the grid the same but varying its diameter, the curves of Figures 25 and 26 were plotted. The rate of variation of the current as a function of the diameter in this case is not as apparent as in the above case, but no doubt careful investigations and a detailed analysis will permit arriving at the establishing of relations between the flow of the current and the diameter of the grid.

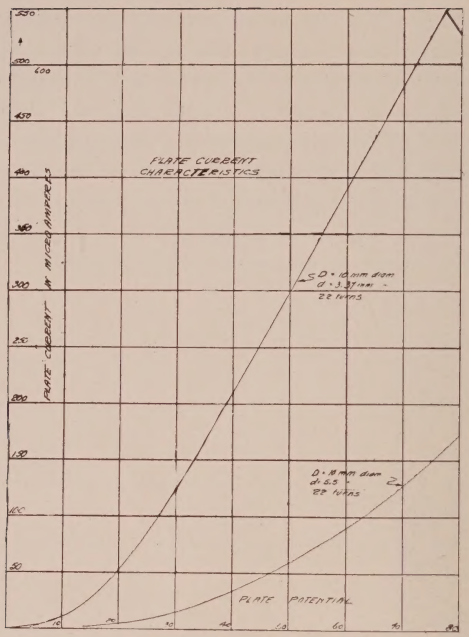



FIGURE 26

Figure 27 shows two curves of tubes having grids and filaments alike in every respect (as near as is possible in their manufacture), but the plate diameters were 8 mm. and 10 mm. (0.316 and 0.394 inch) approximately. In other words, these curves furnish us with information as regards the changes of current as a function of grid-plate distance. With regard to this grid-plate distance, we can state that it appears to have a great influence on the operating characteristics of a tube. As we shall see later, it is this distance that governs the quality of a tube as an amplifier.

We can now take up the study of simultaneous action of all of the three elements of a vacuum tube. Three sets of curves in Figure 4 illustrate completely why it is imperative to duplicate almost all of the foregoing tests. It will be noted from the curves that the three tubes, which, by the way, were taken at random from a large lot, show widely different characteristics. As stated at the very beginning of this article, the causes for such variations are many, and to determine conclusively such causes for the purpose of their elimination, if this be within possibility, it is necessary to carry out series of tests with as many possible

variations of dimensions of elements as will be necessary to complete the mathematical analysis. With the aid of such an analysis the various causes tending to distort the characteristics of a tube could be studied with more precision, as could also the possibilities of determining the best which can be expected of a tube.

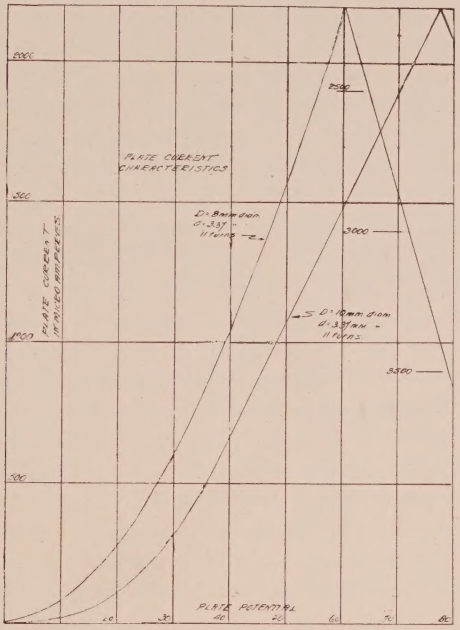



FIGURE 27

Figure 28 shows a complete set of curves of a tube, which by reason of its constancy in action, its characteristics as medium plate-potential amplifier, may be considered a representative type of the vacuum tube. This tube was built for transmitting and amplifying purposes only, and as such has given excellent results. It is the "British B" tube, or Moorhead Type C. These excellent amplifying characteristics were secured by using a 22-turn grid. Comparing with that tube the tube known as "British R" and its duplicate the "SE-1444," we find that by changing the grid from 22 turns to 11 turns, keeping the other elements of the same size, we get a tube which is a fair amplifier

and also a fair detector. The characteristic curves of this type tube are given in Figure 2. The tests of the above tubes, together with series of tests carried out on tubes of various dimensions, bring out the fact that to secure best results in the action of a tube as amplifier a certain location of the grid is required, but that, at the same time, this location of the grid is very unfavorable for detector action.

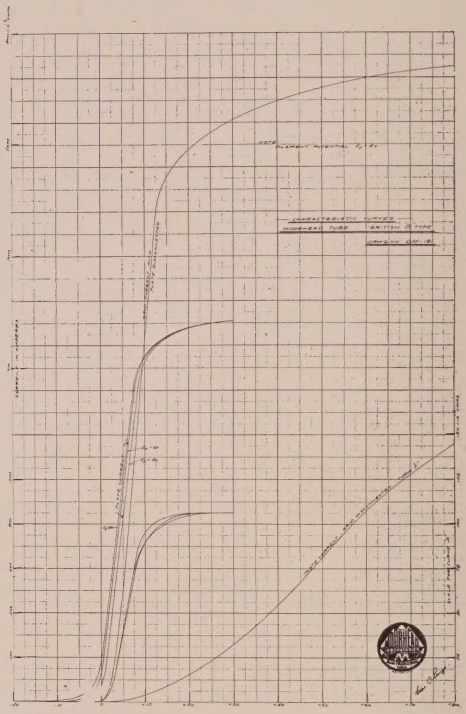


FIGURE 28

The question arises, which conditions must be fulfilled to produce a good detector, and which disposition of elements of the tube is necessary to make the tube a good amplifier. The requirements of a tube as a detector are low internal resistance and that this resistance shall change suddenly, that is, within very narrow limits of grid potential variation. The amplification characteristic of a tube depending on the ratio of the change in

plate potential to the corresponding change in grid potential, the maximum action will be obtained when for a given change of the grid potential the increase in plate potential required to maintain the same current will be a maximum.

Thus we find that in a detector the resistance must drop suddenly, for a small change in grid potential, from a maximum to a minimum; whereas in an amplifier a small change in grid potential should tend to increase the resistance to a maximum. This represents the fundamental difference between the two classes of tubes, and conclusively confirms the statement made by the authors at the beginning of this paper.

Figure 29 gives 4 sets of curves of tubes built specially for determining the action of tubes as detectors and amplifiers. The plates and the filaments of these tubes were made as nearly as possible alike, but the grids were varied; the dimensions chosen were approximately the maximum and minimum considering the diameter of the plate. The exact dimensions and the number of turns of the grid are given in the figure; the standard length of filament and plate, and also the standard pitch of grid windings, were maintained for all four tubes. The results obtained confirmed the theory in every respect.

Inspection of the curves will immediately show that the tubes having 22-turn grids were good amplifiers but poor detectors; the rise of the curves is gradual and the radius of curvature at the knee of the curve, or in the region of operating grid potentials, is comparatively large—on the whole, both tubes proving of inferior value as detectors. Further comparison will show that of the tubes (numbers 13 and 17) number 17 is a better detector than the 13; on the other hand, tube 13 is by far the better amplifier. In other words, the farther the grid is from the filament, and the closer the grid is to the plate, the better are the amplifying qualities of the tube. Referring to the sketch of arrangement of elements in the tube, we may state that for the same number of turns in the grid the smaller is the ratio of $\frac{a}{b}$, the better is the amplification; thus, increasing this ratio will tend to bring out detector qualities at the expense of amplification. This statement holds equally well for the tubes 14 and 15, which are duplicates of 13 and 17 respectively, except for the number of grid turns, which in this case was 11 instead of 22.

Comparing now in pairs tubes 14 and 13, and 15 and 17, we will note that the larger the number of turns in the grid, the

higher the resistance, the better the amplification quality, and conversely.

Picking out of the four tubes the best detector, or tube 15, we find that this tube has the grid of small diameter and small number of turns, or is the opposite extreme of tube 13, which is the best amplifier.

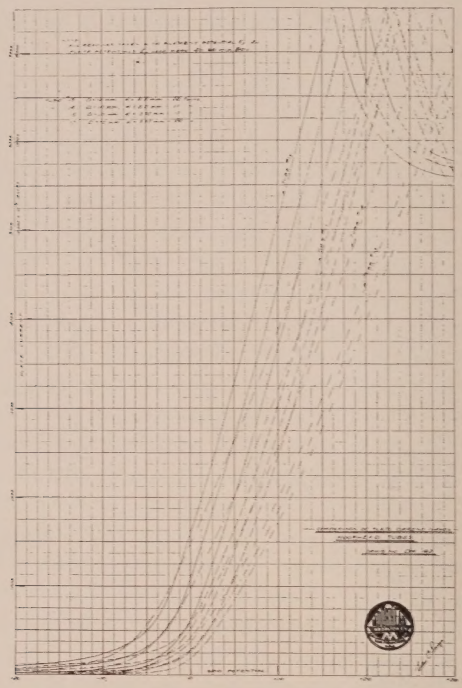


FIGURE 29

Further examination of the tube will reveal an interesting fact, namely that the curves of tubes 13 and 14, or the tubes with grids of larger diameter, do not show saturation within the limits of tests, while the tubes with grids of small diameter, numbers 15 and 17, indicate rather low saturation.

These and many other tests, in the authors' opinion, have proven conclusively that to obtain the best results in the use of tubes in radio, or any other work, at least two different types of

tubes should be employed, selecting proper tubes designed for the purposes intended.

The mathematical formulas expressing the laws of action of tubes, as given to date, do not exactly explain the action, giving one or more constants in their general expression, which constants as indicated by the experiments, are not constants at all, but are variable functions of geometrical dimensions of the tube elements.

SUMMARY: A number of types of Moorhead tubes are described, together with the mode of testing them and the specifications to be met.

The tubes obtained are classified as detectors or amplifiers, which types the authors regard as separate and generally non-inclusive.

The effects of small variations of a number of the tube dimensions are exhaustively studied, and conclusions are drawn as to the effect of varying the various tube dimensions.

# THE CATHODE-RAY OSCILLOGRAPH AND ITS APPLICATION IN RADIO WORK*

By

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The present article is not written with the intention of describing any novel developments in cathode-ray oscillographs or their use, but as a description of the present status of oscillographic practice with special emphasis upon the immense usefulness of the cathode-ray oscillograph in radio research. The work of the Bureau of Standards on this device is described. A bibliography of articles and books with the cathode-ray tube and its applications has been compiled, which is believed to be complete to 1920. Copies of this bibliography can be obtained from the Bureau of Standards, Washington, D. C.

## Essentials of an Oscillograph Tube and Principles of Design

A cathode-ray oscillograph has five essential parts in addition to the glass containing bulb:—(1) a source of electrons which may be either a metal surface subjected to ionic bombardment or a body heated to incandescence; (2) a steady uni-directional potential difference—the accelerating voltage—for giving the electron stream a definite velocity in a straight line; (3) a filtering diafram for the purpose of forming the electron stream into a small beam or pointer; the action of the diafram may be supplemented or improved by the use of a magnetic concentrating field; (4) suitable plates and coils, for impressing upon the beam the electrostatic and magnetic fields, the behavior of which is to be studied; (5) a fluorescent screen for indicating visually the position of the beam. The screen may be coated with any of a number of compounds which fluoresce under electronic bombardment. Willemite ($Zn_2SiO_4$) and calcium tungstate ($CaWO_4$)

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have been found to be particularly suitable. Calcium tungstate glows with a bright blue color under the action of cathode rays, and the traces produced upon such a screen are particularly actinic and suitable for photography. The fluorescent spot formed by the cathode-ray pointer on willemite is greenish yellow, and better suited for visual observation.

Cathode-ray tubes may be divided into two general classes, according to the nature of the cathode, namely, hot-cathode and cold-cathode or Braun tubes. The general principles of design hold for all such apparatus, regardless of the methods employed in obtaining the stream of cathode particles. Approximate expressions relating the sensitivity of the tube, or the deflection per unit of intensity of the deflecting field with the geometrical constants of the tube and the accelerating voltage, can be readily deduced. Consider Figure 1, a diagrammatic sketch of the essential parts of a tube as they are arranged in practice. The

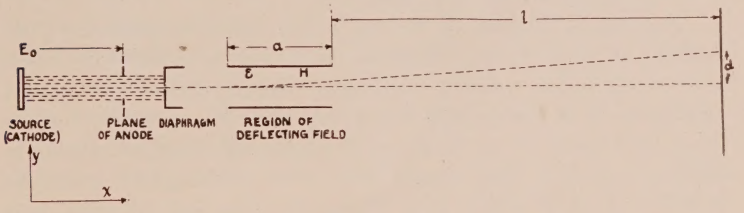


FIGURE 1

steady accelerating voltage $E_o$ (which may, in a cold-cathode tube, be also the *exciting voltage* altho the two functions of this potential difference are separate and independent) acts between the anode and the source. In practice the anode is grounded, and the space between anode and screen is maintained at ground potential, so that the mean velocity of the electrons in the direction of the screen is constant.

Let $x$ be the horizontal co-ordinate parallel with the direction of propagation of the undeviated electrons; let $y$ be the co-ordinate perpendicular to $x$ and measured in the direction of the electrostatic field and perpendicular to the electromagnetic field. Let $a$ be the length of the deflecting field, measured along $x$; $E$ and $H$ the intensities of the electrostatic and magnetic fields, assumed constant for a distance $a$; $l$, the distance from the deflecting field to the screen, and $d_e$, $d_m$ the displacement of the

fluorescent spot on the screen resulting from the action of the electrostatic and the electromagnetic fields, respectively. Then if $v_o$ be the constant velocity imparted to the electrons by the accelerating voltage $E_o$, $e$ the charge on an electron, and $m$ its mass at relatively low velocities, we have the voltage sensitivity of the tube:

$$\xi_e = \frac{a\,l}{2\,E_o\,s} \text{ cm. per volt.}$$

For the deflection in centimeters due to an impressed electromagnetic field of strength $H$ we have:

$$d_m = \frac{H\,a\,l}{\sqrt{2\,E_o}}\sqrt{\frac{e}{m}} \tag{5}$$

These expressions are both based on the assumption that $v_o$ is constant and that the kinetic energy of motion of the electrons is equal to the energy acquired in the potential drop $E_o$. This assumption may be a rash one, if there is a considerable amount of ionizable gas left in the tube.

The magnetic field, $H$ is such a complicated function of the number of turns in the deflecting coils, current in the coils, their size, shape, and spacing, that one cannot define a constant analogous to $\xi_e$ for current sensitivity without such approximations as to make its applications to design calculations extremely limited. Chaffee has stated that with rectangular coils the distance apart of which is $\sqrt{2}$ times the spacing between their planes, the field between the coils is nearly uniform thruout their length. In any case the current sensitivity is increased by increasing the number of turns and the length of the coils. The merit of the tube can be indicated by its voltage sensitivity.

While these two formulas are not entirely rigorous they show that the sensitivity of a given tube for electrostatic and for magnetic deflections increases with the distance over which the deflecting field is applied, the distance between deflecting field and screen, and decreases with increasing value of the accelerating voltage.

The brightness of the fluorescent spot depends upon the number of electrons striking it and upon their velocity, the latter, as we have seen, being directly proportional to the square root of the accelerating voltage, $E_o$. Thus the general problem of design is a compromise between intensity of the fluorescent spot and the sensitivity. Constructional difficulties limit the increase in sensitivity which may be attained by increasing $l$; moreover, in cold cathode tubes there is necessarily left in the tube

enough of ionizable gas to cause the velocity of the electrons to be sensibly diminished as $l$ is increased. Thus, if the purpose for which the tube is to be used determines a maximum value of sensitivity, that is, a maximum value for $E_o$, then the proper brightness of the fluorescent spot must be attained by increasing the only other factor at our disposal, namely, the emission of electrons from the cathode. In a cold cathode tube the emission depends, for a given gas pressure, upon the exciting voltage. But unless the tube is complicated by the addition of an auxiliary anode, the exciting voltage and the accelerating voltage are identical and the emission cannot be be increased without increasing the velocity and decreasing the sensitivity. Thus the construction and use of a cold-cathode tube, the exciting voltage and accelerating voltages of which are the same, is chiefly a matter of experimental manipulation, and very little can be predicted theoretically. The pressure of the residual gas in the tube is an additional important factor. Enough gas must be present to allow the conduction of the exciting current to the anode, but the pressure must be sufficiently low to allow the electrons to pass down the tube without excessive hindrance. Hence, for a given value of the accelerating voltage, dictated by the requirements as to sensitivity, and necessarily determining the exciting voltage, there is a definite optimum value of pressure, for maximum intensity of the fluorescent spot.

The advantage of a tube having a cathode which emits electrons spontaneously is obvious in the light of the foregoing discussion. No exciting voltage is necessary. The rate of emission of electrons depends neither upon the accelerating voltage nor upon the gas pressure. The tube can be exhausted to the lowest possible degree, and the brightness of the spot can be controlled by controlling the temperature of the cathode, the magnitude of the accelerating voltage being determined entirely by the requirements of sensitivity. Thus from the standpoint of design and operation, tubes having a Wehnelt cathode would seem to be the ideal type. However, constructional difficulties involved in the manufacture of hot-cathode tubes are very great. The pressure in an enclosure of several liters capacity must be so low as to allow the electrons when accelerated by only a few hundred volts to travel from 20 to 50 centimeters (8 to 20 inches) in an undeviated path, and must be maintained over long periods of operation.

## Construction of Cold-Cathode Tubes

The difficulties encountered in the construction and operation of two types of tubes are widely divergent and will be discussed separately.

In a cold-cathode tube the pressure, as has been mentioned, is a critical factor in determining the intensity of the cathode beam, and, to some extent, the sensitivity of the tube; hence, it is natural that the vacuum characteristics of the enclosed space should be very important in the construction of the tube. Certain other operational difficulties which were encountered by early investigators have, with the use of improved source of exciting voltage, become less irritating. Zenneck, Roschansky, and Varley who were compelled to use influence machines, were so troubled by electrostatic effects near the cathode and in the vicinity of the screen, both the building up of charges where none were desired, and the leakage of charge between points connected to the terminals of the exciter, that elaborate methods were used in attempts to obtain steady operation. It was found that positive charges collected on the glass near the cathode, causing discharges from the cathode to the glass and increasing the tendency for flash-overs in the tube. Zenneck and Roschansky made the cathodes of various peculiar shapes to minimize these effects, while Minton has stated that the presence of a film of absorbed gas around the cathode, while disadvantageous from the standpoint of maintenance of vacuum, tends to prevent the glass from being charged. This difficulty has been reduced to a minimum in later experiments, with steady sources of high direct voltage, a ground connection upon the anode and upon a tinfoil sheet or wire around the tube in the vicinity of the screen being sufficient to stabilize the conditions of discharge and electron flow down the tube.

In general it is observed that in cold-cathode tubes in which the vacuum is permanent, the pressure decreases with use. Rankin has shown that the hardening of such a tube with use is accentuated by the use of a strong magnetic concentrating field. On the other hand, tubes which are exhausted without certain precautions which insure the increase of vacuum after sealing off, soften with use, sometimes quite rapidly. There is no middle ground; it appears to be impossible so to exhaust a cold-cathode tube that the vacuum will remain constant thruout long periods of operation. In view of the fact that the vacuum cannot be increased by external means, the lesser evil is to exhaust the tube by such means as to produce an increasing vacuum, and to in-

clude a regulator tube of palladium or platinum in order to introduce minute quantities of air when the vacuum tends to increase. It has been demonstrated by Minton in conclusive experiments that the softening of a tube with use is due to the adsorption of a thin film of air over the surface of the glass and of the electrodes. Even if the tube be exhausted to less than a micron, that is, 0.001 mm. of mercury (the normal operating pressure is 3 to 10 microns) at atmospheric temperature, the passage of the discharge for only a few moments will release enough of this *adsorbed* film to raise the pressure above the best operating value. Hence it is essential that enough of the film be driven off during exhaustion that the pressure will not rise noticeably with use. This may be accomplished by heating the whole tube to a temperature just below the collapsing point of the glass and by operating the tube vigorously thruout the exhaust process. Without the heating during exhaustion, the air film invariably remains in the tube and causes trouble later. This process should not be confused with the heating of the electrodes to incandescence, which is done in an electron tube during exhaustion, and which is for the purpose of freeing the electrodes of *absorbed* gases. The electrodes in a cathode-ray tube are never subjected to high temperatures during use, and hence there is very slight possibility of the liberation of *absorbed* gases from them.

The cold-cathode tubes constructed at the Bureau of Standards are of the type shown in the diagrammatic sketch, Figure 2. A polished concave aluminum cathode is used, with radius of curvature somewhat less than the distance to the screen. The diafram and anode are combined in one piece and made of brass. A single aperture 0.025 cm. (0.01 inch) in diameter is used, the piece being built in two parts with a shielded opening around the outside edge, in order to allow the free passage of gas back and forth during exhaustion. Thin brass plates 8 cm. (3.4 inches) long, for electrostatic deflections were sealed into one tube. It was eventually found desirable, however, to facilitate exhaustion by reducing the amount of metal in the enclosure to a minimum. The gain in sensitivity resulting from a closer spacing between the plates does not justify their enclosure in the tube unless the tube be constructed for some special purpose. Screens of soda glass formed into circular plates were used instead of mica. The powdered calcium tungstate was sifted upon the surface which had previously been coated with china painters' varnish and the whole was heated until the powder became

imbedded in the glass. The substitution of glass for mica is an expedient of doubtful advantage because the glass is much heavier and must be wired in place thru holes around the edge, to supporting posts which have previously been sealed inside the containing bulb. The drilling process usually starts minute cracks which speedily ruin the plate when it is heated, regardless of the care used in handling. The tubes proper were con-

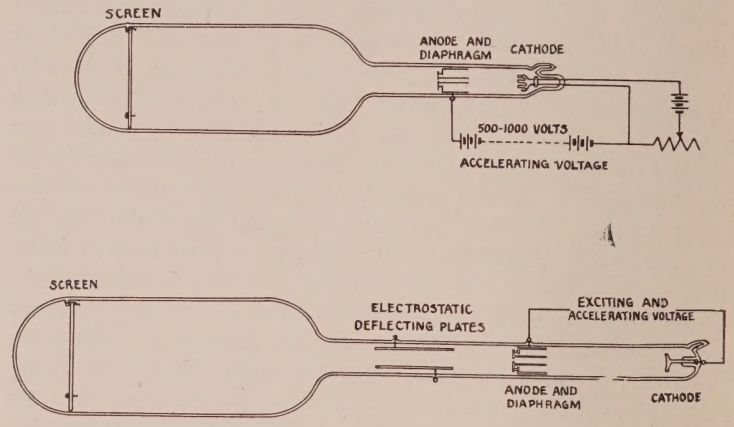


FIGURE 2

structed of soft soda glass, which could be readily worked, and which has been found by Minton to be less conducive to adsorption of air than lead glass. The several parts of these tubes (fluorescent screens, aluminum cathodes, and brass anodes) were designed and constructed by the General Electric Company, and were purchased from the Pittsfield research laboratory of that Company.

A Langmuir condensation pump in cascade with an oil-sealed rotary pump was used in the exhaustion. During exhaustion the tubes were kept at a temperature of 350° to 400° Centigrade in a specially constructed electric oven, and a heavy discharge was maintained between the anode and cathode. The chief reason for maintaining this discharge at known voltages was to indicate the condition of the vacuum. The assistance rendered by the discharge in reducing the vacuum is practically negligible, as Minton has found that in tubes exhausted while the discharge is maintained without heating, the vacuum invariably softens with use.

The electric oven used is made of galvanized iron and lined with quarter-inch asbestos board. It encloses a space four feet (1.2 m.) long by one foot (0.31 m.) square, which is maintained at a temperature of 400° Centigrade by twelve coils of "nichrome" wire so connected as to dissipate about 4 kilowatts at 110 volts.

The exhaustion of the tubes in this oven is rather a slow and laborious process owing to their large volume and to the large amount of enclosed metal. The high voltage discharge gives a sufficient indication of the progress and condition of the vacuum. Usually the exhaustion is far from complete when the Crookes dark space expands and evidence of cathode rays first appear. For a given exciting and accelerating voltage a definite optimum pressure is reached, as judged by the intensity of the fluorescent spot. This occurs when the positive column extends out from the cathode over a distance of from 0.5 to about 3 centimeters (0.2 to 1.2 inch). It is well to operate the tube heavily at this pressure before separating it from the pump.

### Excitation of Cold-Cathode Tubes

By no means the least of the disadvantages of a cold-cathode tube employing the excitation method of obtaining electrons is the high direct voltage required. A voltage of 5,000 is the extreme minimum which has been found could be used in practice with a soft tube. If the gas pressure be high enough or higher than enough to allow an exciting voltage of 5,000 to produce a reasonably dense electron stream, there is danger of flash-overs, and the whole tube glows with ionized gas to such an extent as to make photography difficult. It has been found in practice that 10,000 to 20,000 volts are good compromise values. An exciting voltage of 10,000 together with the optimum degree of vacuum will provide enough electrons to produce a bright spot on the screen; the pressure for best production and propagation of electrons is such that the positive column is only a few centimeters long and the Geissler effects in the tube are not particularly brilliant. This value of accelerating voltage gives a theoretical voltage sensitivity of 0.011 cm. per volt in a 35-centimeter (13.6 inch) tube with 10 cm. (3.9 inches) deflecting plates spaced 1.5 cm. (0.59 inch) apart.

Few laboratories are equipped with storage cells enough to supply 10,000 volts, which would be the ideal source of supply, and other methods must be employed. Following the example of the early investigators, Ryan, Fleming, Dufour, and others

have advocated the use of influence machines. In the first place, the only possible advantage attached to the use of an influence machine is the fact that the voltages supplied may be as high as 100,000 volts, and extremely hard tubes can be excited. If one wishes to measure dielectric power losses at high voltage it is necessary to use a hard tube of low sensitivity, and consequently the high exciting voltages are necessary. For ordinary oscillograph work, in which tubes of high sensitivity are required, it is essential that the accelerating voltage be kept low, and the electrostatic effects about the cathode, leakage, and general erratic behavior of an influence machine make it quite unsuitable. Moreover, after the shielding and drying precautions already mentioned are taken, a Wimshurst machine of sufficient number of plates and careful enough design to supply the necessary 50 to 500 micro-amperes without exceedingly vexatious and troublesome variations in the terminal voltage is about as uncommon as a 10,000-volt storage battery.

Minton has successfully used a mechanical rectifier with high-voltage alternating currents. It has been found at the Bureau of Standards that electron tubes are the ideal rectifiers for this purpose. Excellent two-electrode rectifying tubes ("kenotrons") are manufactured in this country by the General Electric Company, and the circuit shown in Figure 3 has been used as an exciting circuit. This system was first described by

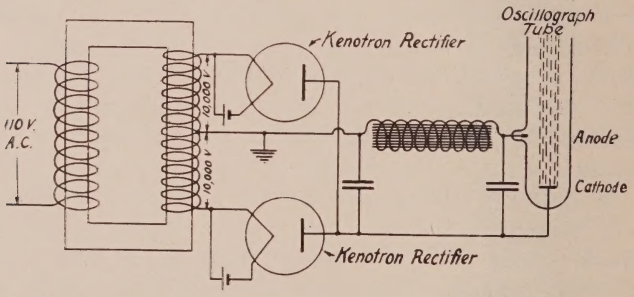


FIGURE 3

A. W. Hull ("Physical Review," March, 1916, page 405). A 110-30,000 volt transformer with a lead-in to the middle of the secondary winding is used at $T$. $C$ is 0.04 $\mu$f. and $L$ an iron-core choke coil of about 5 henrys inductance. The apparatus

can be mounted on a suitable panel with separate transformers for the high-voltage supply and filament supply, forming a portable high voltage d. c. source which can be operated from 60-cycle mains. The power that can be taken from the rectifier, which is not of much importance in a cathode-ray tube, is limited only by the size of the kentrons used.

## Construction of Hot-Cathode Tubes

Altho, as has been stated, the idea of utilizing a heated cathode as a source of electrons, thus making the supply independent of the accelerating voltage and easily controllable, was first conceived some time ago, the applications of this principle have been very limited. The difficulties are not in the conception nor in the theory, but in the construction, and so this discussion will be concerned chiefly with constructional details.

In a tube having an independently controllable source of electrons, it is not desirable to reduce the pressure to a certain optimum point of a few microns and then maintain it there, but to keep it at the lowest possible point all the time, since the electron flow is never assisted but always hindered by the pressure of gas. This was realized by Crooker who was compelled to keep his tube on the vacuum pump continually during his investigations. Perhaps the most successful work done in this country with hot-cathode tubes has been performed at Harvard University by Dr. Chaffee, Mr. R. F. Field, and their associates. They utilized tubes already manufactured by Muller-Uri, cutting off the constricted portion of the tube containing the platinum and aluminum electrodes, and utilizing the light, excellently coated and mounted fluorescent screens which are the chief merit of these German tubes. They used an anode made in the form of a ring, of diameter just small enough to go into the tube, and no filtering diafram whatever, depending entirely upon concentrating coils having a large number of ampere-turns to focus or concentrate the electron stream. The cathode is a thin strip of platinum, perhaps a millimeter (0.04 inch) wide, having a deposit of Wehnelt oxides about the size of the head of a pin deposited on the surface, the idea being to limit the emitting surface and depend upon focussing, rather than to provide a large supply of electrons and filter out the excess with a diafram with the possibility of increasing their concentration in the indicating ray. Being rather small, these tubes are comparatively easy to exhaust, and a fine sharp point is obtained on the fluorescent screen.

The first experiments along this line at the Bureau of Standards were performed with an experimental tube of small diameter, which could not have been used for oscillographic purposes. This tube was quite long, but the volume was small, in order to allow its rapid exhaustion. It was kept on the pump continuously and was so constructed as to expedite the removal of the cathode, in order to test different substances. The anode and diafram were separate, the anode being a thin ring of soft iron, and the diafram being also on an iron frame. The anode was connected to the sealed-in contact with a spiral of fine wire, the plan being to move the anode and diafram back and forth with a magnet after exhaustion of the tube, and determine, in connection with two concentrating coils of 250 turns each, placed around the tube, the optimum distance from screen and anode to diafram.

Tungsten and oxide-coated platinum cathodes of various forms were tried in this tube, the best type being a W-shaped platinum filament coated with barium and calcium oxides. Such a filament could be lighted to full brilliancy by a single lead cell. Tungsten filaments without an oxide coating have been used extensively by Coolidge in Rontgen-ray tubes, and Samson has decided that such filaments are useful in oscillograph tubes. However, it has been found that oxide-coated filaments give a much more bountiful supply of electrons, with less light, owing to the low filament temperature. As far as the vacuum characteristics are concerned, it is found that filaments coated with the composition used so successfully by the Western Electric Company in their electron tubes do not emit gases to a troublesome extent. Numerous experiments with varying distances between the electrodes, values of accelerating voltage, and configurations of the concentrating fields were very interesting. The best pressure for such work is considerably greater than the pressure maintained permanently in a tube. If there be gas enough present so that the electron stream ionizes the air on passing thru, the presence of the beam can be detected as a luminous blue streak, and can be much more easily focussed and adjusted. The effects of the concentrating coils can be seen at a glance. The bundle of "rays" emanating from the filament cathode and made visible by the ionized air can be formed into beautiful spiral figures by the action of the focussing coils. It was found, in general, that a concentrating field when used to best advantage is close to the diafram, and remote from the cathode. Apparently an ideal field for this purpose is one

in which the lines of force are rectilinear and accurately parallel with the axle of the tube. If the lines of magnetic force are not parallel with the axis of the tube, the resultant force upon the moving electron due to its velocity and to the perpendicular component caused by the magnetic field will deflect its spiral path away from the axis of the tube as an axis of rotation and will cause the lines on the screen to depart from proportionality with the transverse fields.

The convolutions of the spiral path of the electron stream grow narrower as the field strength is increased. It seems to make little difference in the intensity of the spot whether the field is applied before the electrons encounter the diafram, directing the concentrated beam at the aperture in the diafram or after the small bundle of rays leaves the diafram.

The use of two focussing coils is superior to the use of one only in the matter of ease of adjustment. The same intensity and brightness of the indicating spot can be obtained eventually with one, as with two. It was found, moreover, that for a given accelerating voltage, the spacing between anode and diafram seemed to make no particular difference. An arrangement of concentrating coils for maximum brightness could be found for almost any position and spacing of the electrodes. It was finally decided that the simplest possible arrangement was quite adequate, that is, the use of one concentrating coil of about 500 ampere-turns, and a single electrode as diafram and anode, as in the cold cathode tubes. No special advantage was discovered in the experimental tube by having the anode and diafram separate. The following are the dimensions of a completed tube: screen to anode-diafram, 38 cm. (14.8 in.); anode-diafram to cathode, 11.4 cm. (4.44 in.); tube 3.2 cm. (1.25 in.) and 13 cm. (5.1 in.) in diameter; cathode about 0.6 cm. (0.23 in.) by 0.6 cm. (0.23 in.). A diagrammatic sketch of this tube is shown in Figure 2. The aperture in the diafram is only 0.027 cm. (0.01 in.) in diameter, and with the concentrating coil in the most favorable position a sharp spot about one millimeter (0.04 in.) in diameter is seen upon the screen. When used with an accelerating voltage of 500 volts a voltage sensitivity of about 0.25 cm. per volt has been obtained with this tube.

## Applications to Radio Research

The cathode ray oscillograph has been used in the radio laboratory of the Bureau of Standards in general testing work and also as an almost indispensable aid to research on radio

frequency phenomena. It is employed for three different purposes: (1) to trace radio frequency current and voltage wave forms; (2) to trace the volt-ampere characteristics of a given device or network; (3) to synchronize radio frequency circuits.

The oscillograph cannot be used to trace voltage and current wave forms without apparatus accessory to that which is being investigated, since a time axis is always required. That is, if the voltage or current deflection is vertical, means for deflecting the spot horizontally at some lower frequency are necessary, in order that the curve be developed along an axis upon which the displacement is proportional to time or to some known function of time. Moreover, the vibration producing the time deflection must be accurately synchronized with the current or voltage oscillations, because the end of the cathode beam impinging on the screen may be traveling with a velocity of thousands of meters per second, and it must traverse exactly the same path in order to cause the screen to fluoresce. This synchronization of the time deflection with the current or voltage deflection is frequently a difficult matter, particularly where apparatus is under the test of which the behavior must not be altered by the influence of the measuring devices. When put to the uses (2) and (3) mentioned above, there is no such difficulty of synchronization, but careful design of the deflecting coils and plates is necessary in order to produce sufficient deflections without changing the constants of the device or network which is being investigated.

In Figures 4 and 5 are shown oscillograms taken as part of a test of a quenched spark set of unique construction submitted to the Bureau of Standards for test. The first five pictures show volt-ampere characteristics of the quenched gap when operating under different adjustments at a frequency of 500,000 (600 meters). Voltage across the closed circuit capacity is traced horizontally and current thru the closed oscillatory circuit is traced vertically, producing a fair approximation of a highly damped logarithmic spiral under best operating conditions. In the remaining six figures are shown current wave forms in the antenna circuit at 600 meters, for various adjustments of coupling and capacity in the closed circuit. The time axis is obtained in all cases by connecting the voltage deflecting plates across the primary condenser. The variation in voltage which develops the radio frequency wave train is not the 500-cycle supply voltage, but the rapid voltage rise as the condenser is charged, which takes place several times at the peak of the audio frequency

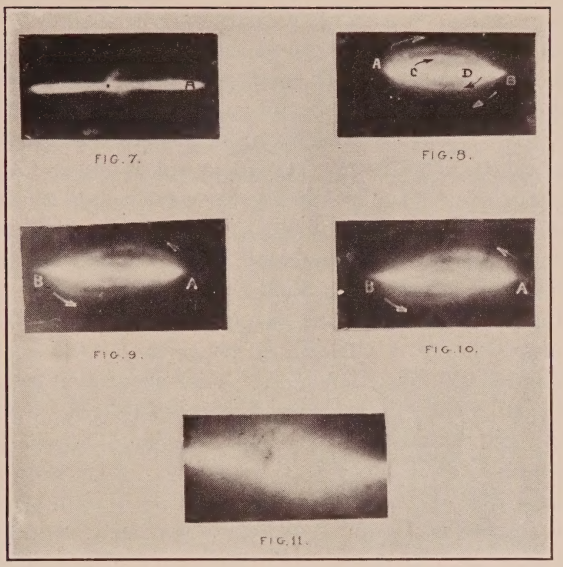


FIGURE 4

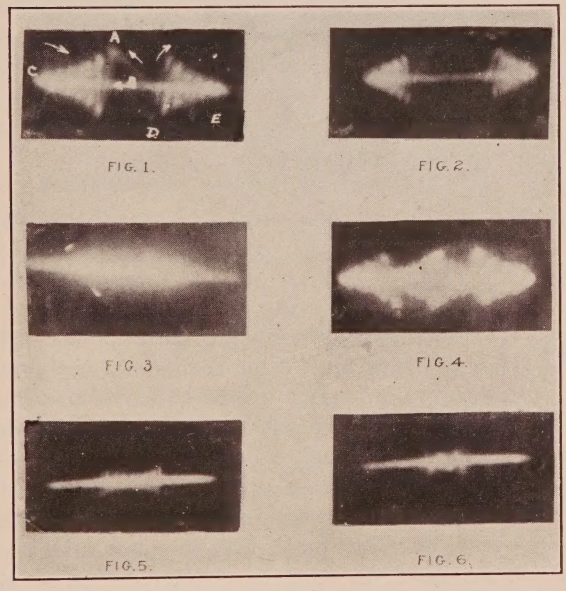


FIGURE 5

cycle, and which correspond roughly to a frequency of 10,000 per second. It is true that these curves do not have the neat and clear-cut appearance of similar cathode ray photographs that have been published from time to time. It should be understood, however, that they are given as an indication of what can be done with a cathode ray tube as an aid to routine testing of spark apparatus which is admittedly erratic and uncertain in its behavior. It is not intended to portray the regular and consistent behavior of a well-designed spark set, but to show the possibilities in investigating faulty spark transmitters.

In Figure 6 are shown the envelopes of the antenna current of an electron tube transmitting set with alternating-current plate supply voltage, designed at the Bureau of Standards. The horizontal deflection is caused by the 500-cycle plate supply voltage, is sinusoidal and hence does not spread out the 300,000-cycle antenna current sufficiently to show the separate oscillations. All that was desired in this case was an indication of the antenna current envelope, which could not have been obtained by any other oscillograph without the use of a rectifier.

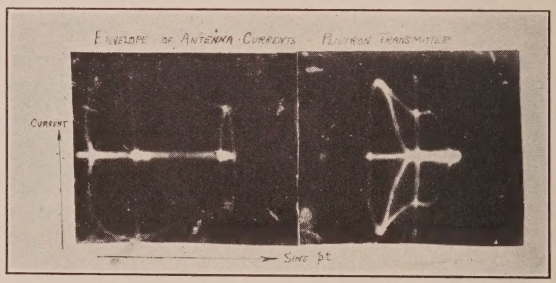


Figure 6

In Figure 7 are shown plate-current, grid-voltage characteristics of a lower power electron tube generator supplying about 3 watts output at wave lengths of 500 to 2,500 meters in an inductively coupled circuit. These diagrams were made some time ago, with a cold-cathode tube, and the figures were not sufficiently intense to be photographed. They were sketched on co-ordinate paper to correspond to co-ordinates ruled on the fluorescent screen. Cold-cathode tubes are, as a rule, too insensitive to obtain good volt-ampere characteristics of electron tubes, particularly when the space currents are involved. The

plate and grid currents, being usually less than 0.05 ampere in amplitude, require deflecting coils of such magnitude as to affect and alter materially the characteristics of the output circuit. This is shown in these diagrams. The figures obtained at the longer wave lengths agree closely with the low frequency and static characteristics of the tube. However, as the frequency is increased, the reactance of the series deflecting coils in the plate circuit becomes comparable with the effective resistance of the output circuit and the plate current lags behind the grid voltage.

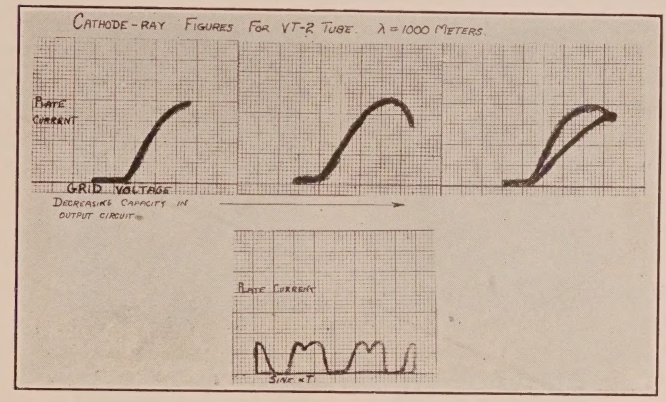


FIGURE 7

This may be complicated in practice by another effect, the results of which do not appear here, namely the distortion of the plate-current wave from its normal form as the deflecting coils become resonant to, and tend to choke out the various harmonics. In all generating circuits the load on the tube is a parallel resonance load, with the result that the harmonics of current from the tube due to saturation and rectification effect flow from the tube into the output system practically unchanged, maintaining a highly flattened and distorted wave form of space current, while the harmonics of voltage are choked out in the output system. Where a series reactance in the form of a deflecting coil is inserted in the circuit, its impedance to frequencies as high as the 5th harmonic of the radio frequency fundamental may be large compared with the 3,000 to 10,000-ohm tube resistance with the result that the normal operation

of the system is altered by the elimination of the current harmonics.

At the present stage of development of the oscillograph it is impossible to obtain radio frequency detector and amplifier characteristics, since the operating range of such devices involves millivolts and microamperes, associated with high impedances. The only known method of determining dynamically the behavior of small tubes is by the use at audio frequencies of an Einthoven, or some other form of vibration galvanometer.

In Figure 8 are shown some current wave forms of an electron tube generator operating at about 3,000 meters. The strong

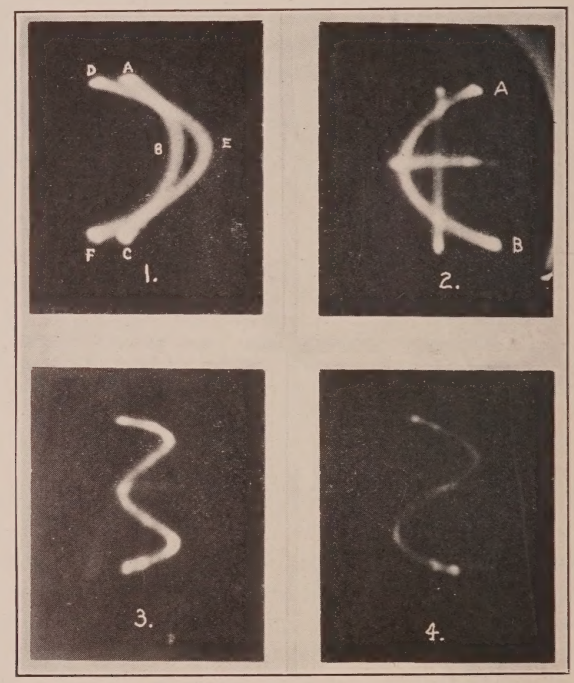


FIGURE 8

harmonics in the space of the lower diagram of Figure 7 and the apparent absence of harmonics in the circulating output current are plainly evident (shown in Figure 8).

Deflections along the horizontal axis in these photographs are proportional to the sine of time, being produced by the alternating voltage across the condenser of a separate generating

circuit which was tuned exactly to a lower frequency, a submultiple of the 500,000-cycle current, the wave form of which is being traced. Thus the figures are merely Lissajous figures in which the higher frequency is distorted by harmonics and is an integral multiple of the lower frequency. It is easy to trace from these photographs, by measuring properly spaced ordinates, the true current-time curves.

Beautiful and intricate Lissajous designs can be formed with two electron tube generating circuits and any cathode-ray tube in which the spot is bright and sharp. The ease with which the fundamental frequencies of such circuits can be continuously varied makes their use in this connection a fascinating as well as a profitable study. It has hitherto been considered possible to syntonize circuits only by resonance methods. This can be done with considerable accuracy using continuous waves, since measuring circuits of extremely low resistance can be employed. But the best accuracy which has been attained in comparing standard circuits at 600 meters, for example—is about 0.2 per cent., which means a discrepancy in frequencies of 500 cycles per second. It has even been a matter of conjecture whether electron-tube generators would maintain greater constancy of frequency than this from one minute to the next. Using the cathode-ray tube, however, it is possible to synchronize generating circuits to an accuracy of a small fraction of a cycle per second out of 500,000. In Figure 9 are shown copies of photographs of the figures obtained at random tuning two circuits to various frequency ratios. Deflections from the sinusoidal output current in one circuit are impressed against the equally sinusoidal output voltage from another circuit. Of course it is necessary to take elaborate precautions of grounding and shielding the coils and condensers from each other and from surrounding objects in order to attain nearly perfect syntonization. Even then the final adjustments can be readily made by changing the position of the operator's hand with respect to any ungrounded terminals, or about the generating tube itself, if the latter be left unshielded. A slight detuning of one of the circuits causes an apparent rotation of the figure about an axis in the plane of the photograph and parallel to the axis along which deflection at the lower of the two frequencies is taking place. One revolution of the figure per second corresponds to a frequency difference of one cycle; extremely careful shielding of the circuits from each other is necessary to prevent their pulling into synchronism and phase.

Such experiments are of interest in showing the constancy with which an electron tube oscillator will maintain a given radio frequency. If storage batteries be used to heat the filaments and to supply the plate circuits, and the tubes be allowed to operate for some time in order to become stabilized, the shielded circuits can be adjusted to a given ratio of frequency

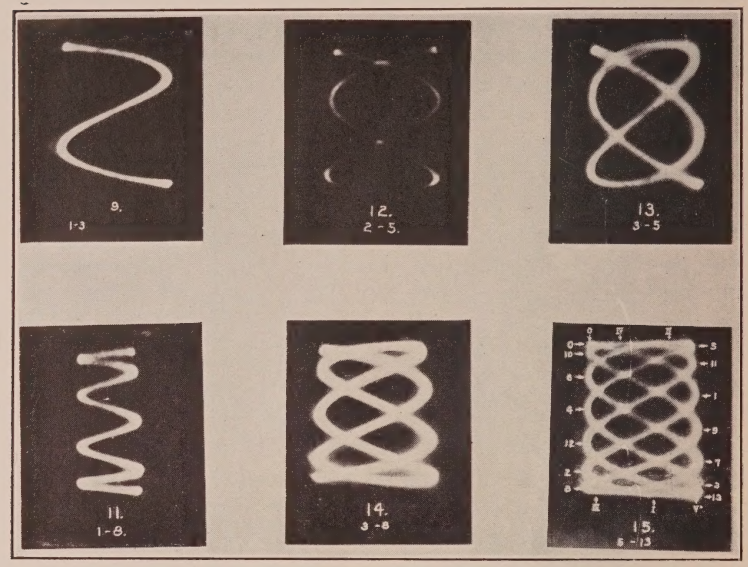


FIGURE 9

and left for minutes at a time without deviating as much as one cycle per second from these frequencies.

It becomes immediately apparent that radio frequency measurements which are performed by resonance methods can be performed with greater refinement by the use of an oscillograph for comparing frequencies. Consider the measurement of the distributed capacity of coils at radio frequencies. When using fixed inductance and fixed capacity in one circuit, with a standard variable condenser in the other, it was found that if the frequency of the variable circuit be increased from one ratio to another the capacity does not increase as the square of these ratios. For example, if the fixed circuit oscillates at a frequency $n$ and if the variable circuit be tuned with a capacity $C$ to a

frequency $n$, the capacity for frequency $2n$ is slightly less than $4C$, the capacity for frequency $3n$ is less than $9C$, and so on. These discrepancies are due to the distributed capacity of the inductance in the variable circuit, and from two settings of the condenser the absolute value of this extremely small capacity can be calculated with greater accuracy than is possible by ordinary methods employing a wavemeter, using well-known formulas. ("Circular of the Bureau of Standards," number 74, page 101.)

The usefulness of the cathode-ray oscillograph in radio measurements is chiefly qualitative. Except when it is employed as an aid to frequency comparison between two circuits, precision measurements cannot be made, as the deflections can not be accurately measured when photographed. It is as an aid to research, permitting visual observations of phenomena hitherto unseen and furnishing qualitative data for new ideas and new theories, that the cathode-ray oscillograph performs a service that can be achieved by no other device.

Washington, D. C., February 20, 1920.

SUMMARY: The general construction and details of the design of cathode ray tubes are given. The brightness of the spot obtained, the sensitivity of the tube, and its behavior in use are described. The various excitation sources available are compared. A number of applications of such tubes to radio research (for example, spark gap characteristics, and associate synchronizing of oscillating triodes) are described.

# FREQUENCIES AND DAMPING FACTORS OF COUPLED CIRCUITS*

By

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In nearly all of the many investigations of the subject of coupled circuits, the discussions were largely limited to the evaluation of the frequency constants and in some cases the determination of the current amplitudes; also only magnetic coupling was generally considered. The matter of damping factors was generally ignored, and the reason for it is the mathematical complications introduced in any attempt to obtain the complete solution of coupled circuits.

In the study, however, of oscillatory currents it is frequently of great importance to have some knowledge of the damping factors; it may be very helpful in investigations relating to oscillatory currents due to local disturbances or static effects.

In this paper a comparatively simple and effective method is developed which is applicable for the determination of the frequency and damping constants of any electrical system of two degrees of freedom, and the work is given in detail for the cases of magnetic coupling, direct coupling and electrostatic coupling.

## Magnetic Coupling

The circuit equations are as follows:

$$\left.\begin{aligned} L_1\frac{d^2I_1}{dt^2}+R_1\frac{dI_1}{dt}+\frac{I_1}{C_1}+M\frac{d^2I_2}{dt^2}=0 \\ L_2\frac{d^2I_2}{dt^2}+R_2\frac{dI_2}{dt}+\frac{I_2}{C_2}+M\frac{d^2I_1}{dt^2}=0 \end{aligned}\right\} \quad (1)$$

Assume $I_1=A\varepsilon^{\lambda t}$, $I_2=B\varepsilon^{\lambda t}$ and substituting in (1), we get

$$\left.\begin{aligned} (L_1C_1\lambda^2+R_1C_1\lambda+1)\,A+MC_1\lambda^2B=0, \\ (L_2C_2\lambda^2+R_2C_2\lambda+1)\,B+MC_2\lambda^2A=0. \end{aligned}\right\} \quad (2)$$

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Eliminating $\frac{A}{B}$ we get a fourth degree equation as follows:

$$C_1 C_2 (L_1 L_2 - M^2) \lambda^4 + C_1 C_2 (L_1 R_2 + L_2 R_1) \lambda^3 + (L_1 C_1 + L_2 C_2 + R_1 R_2 C_1 C_2) \lambda^2 + (R_1 C_1 + R_2 C_2) \lambda + 1 = 0. \quad (3)$$

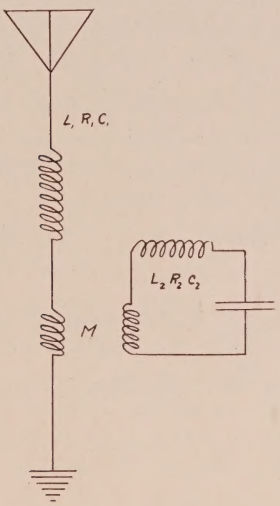


FIGURE 1

Since the currents are of damped oscillatory character, $\lambda$ is a complex quantity, the real term the damping factor, and the imaginary term the frequency constant of the system. Putting $\lambda = \alpha + j\beta$, we get

$$\lambda^2 = \alpha^2 - \beta^2 + j\,2\,\alpha\,\beta,$$
$$\lambda^3 = \alpha^3 - 3\,\alpha\,\beta^2 + 3\,j\,\alpha^2\beta - j\,\beta^3,$$
$$\lambda^4 = \alpha^4 - 6\,\alpha^2\,\beta^2 + \beta^4 + j\,4\,\alpha^3\,\beta - 4\,j\,\alpha\,\beta^3.$$

In all high frequency circuits $\alpha$ is small in comparison with $\beta$, and by neglecting $\alpha^2$ in comparison with $\beta^2$ the error introduced is extremely small, so that to a very high degree of approximation we may write,

$$\left.\begin{aligned} \lambda &= \alpha + j\beta, \\ \lambda^2 &= -\beta^2 + 2\,j\,\alpha\,\beta, \\ \lambda^3 &= -3\,\alpha\,\beta^2 - j\,\beta^3, \\ \lambda^4 &= \beta^4 - 4\,j\,\alpha\,\beta^3. \end{aligned}\right\} \quad (4)$$

Substituting the values of $\lambda$ from (4) into (3), we get

$$C_1 C_2 (L_1 L_2 - M^2)(\beta^4 - j\,4\,\alpha\beta^3) - C_1 C_2 (L_1 R_2 + L_2 R_1)(3\,\alpha\beta^2 + j\,\beta^3) + (L_1 C_1 + L_2 C_2 + R_1 R_2 C_1 C_2)(-\beta^2 + j\,2\,\alpha\beta) + (R_1 C_1 + R_2 C_2)(\alpha + j\,\beta) + 1 = 0. \quad (5)$$

Separating the reals and imaginaries, we have

$$C_1 C_2 (L_1 L_2 - M^2)\beta^4 - \{3\,\alpha C_1 C_2 (L_1 R_2 + L_2 R_1) + L_1 C_1 + L_2 C_2 + R_1 R_2 C_1 C_2\}\beta^2 + (R_1 C_1 + R_2 C_2)\alpha + 1 = 0, \quad (6)$$

$$-4 C_1 C_2 (L_1 L_2 - M^2)\alpha\beta^2 - C_1 C_2 (L_1 R_2 + L_2 R_1)\beta^2 + (L_1 C_1 L_2 C_2 + R_1 R_2 C_1 C_2)\,2\alpha + (R_1 C_1 + R_2 C_2) = 0. \quad (7)$$

The resistance terms in equation (6) are very small in comparison with the other terms, hence to a very high degree of approximation equation (6) reduces to

$$C_1 C_2 (L_1 L_2 - M^2)\beta^4 - (L_1 C_1 + L_2 C_2)\beta^2 + 1 = 0 \quad (8)$$

and

$$\beta^2 = \frac{L_1 C_1 + L_2 C_2 \pm \sqrt{(L_1 C_1 - L_2 C_2)^2 + 4 M^2 C_1 C_2}}{2 C_1 C_2 (L_1 L_2 - M^2)}. \quad (9)$$

For resonance condition, $L_1 C_1 = L_2 C_2 = LC$, equation (9) reduces to

$$\beta^2 = \frac{2LC \pm 2M\sqrt{C_1 C_2}}{2L^2 C^2 \left(1 - \frac{M^2}{L_1 L_2}\right)},$$

$$= \frac{1}{LC(1 \mp \rho)}, \quad (10)$$

where $$\rho = \frac{M}{\sqrt{L_1 L_2}}.$$

The values of $\beta^2$ given by (9) and (10) are the well-known expressions for the frequency constants of magnetically-coupled circuits; $\rho$ is the coupling coefficient.

By equation (7), we have

$$\{2 L_1 C_1 + 2 L_2 C_2 + 2 R_1 R_2 C_1 C_2 - 4 C_1 C_2 (L_1 L_2 - M^2)\beta^2\}\alpha = C_1 C_2 (L_1 R_2 + L_2 R_1)\beta^2 - R_1 C_1 - R_2 C_2$$

and

$$\alpha = \frac{C_1 C_2 (L_1 R_2 + L_2 R_1)\beta^2 - R_1 C_1 - R_2 C_2}{2 L_1 C_1 + 2 L_2 C_2 + 2 R_1 R_2 C_1 C_2 - 4 C_1 C_2 (L_1 L_2 - M^2)\beta^2}. \quad (11)$$

Introducing in (11) the values of $\beta^2$ from (9) or (10), we get the complete expressions of the damping factors expressed in terms of the electrical constants of the circuits. In any given

case it is simpler to obtain first the numerical values of $\beta^2$ and substituting in (11) to obtain the values of the damping factors. For the resonance condition, a considerable simplification of (11) can be obtained. We have

$$C_1 C_2 (L_1 R_2 + L_2 R_1) \beta^2 = \frac{C_1 C_2 (L_1 R_2 + L_2 R_1)}{L C (1 \pm \rho)}$$

$$= \frac{R_2 C_2 + R_1 C_1}{1 \mp \rho},$$

$$4 C_1 C_2 (L_1 L_2 - M^2) \beta^2 = \frac{4 L^2 C^2 (1 - \rho^2)}{L C (1 \mp \rho)} = 4 L C (1 \pm \rho).$$

Substituting above values in (11), we get

$$\alpha = \frac{(R_2 C_2 + R_1 C_1) \left( -1 + \frac{1}{1 \mp \rho} \right)}{4 L C + 2 R_1 R_2 C_1 C_2 - 4 L C (1 \pm \rho)},$$

$$= \frac{\pm \rho (R_2 C_2 + R_1 C_1)}{(2 R_1 R_2 C_1 C_2 \mp 4 L C \rho)(1 \mp \rho)}. \qquad (12)$$

Equation (12) is the complete expression for the damping factors of magnetically-coupled circuits at resonance condition. In all practical cases $R_1 R_2 C_1 C_2$ is very small in comparison with the term $L C \rho$, and equation (12) simplifies still further to the following:

$$\alpha = - \frac{\left( \frac{R_2}{4 L_2} + \frac{R_1}{4 L_1} \right)}{1 \mp \rho}. \qquad (13)$$

Direct Coupled Circuits

The circuit arrangement is shown in Figure 2 and the circuit equations are as follows:

$$\left.\begin{aligned} L_o \frac{d^2 I_o}{dt^2} + R_o \frac{d I_o}{dt} + \frac{I_o}{C_o} + L_2 \frac{d^2 I_2}{dt^2} + R_2 \frac{d I_2}{dt} = 0, \\ L_o \frac{d^2 I_o}{dt^2} + R_o \frac{d I_o}{dt} + \frac{I_o}{C_o} + L_1 \frac{d^2 I_1}{dt^2} + R_1 \frac{d I_1}{dt} + \frac{I_1}{C_1} = 0. \end{aligned}\right\} \qquad (14)$$

Also,

$$I_o = I_1 + I_2.$$

Substituting the above value of $I_o$ in (14) and rearranging, we get

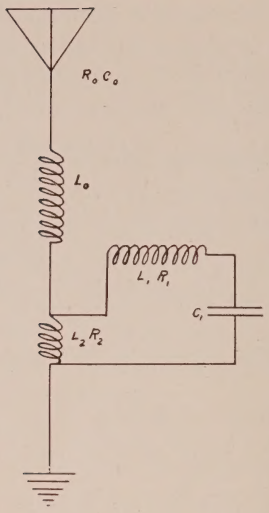


FIGURE 2

$$\left.\begin{aligned} &L_o\frac{d^2I_1}{dt^2}+R_o\frac{dI_1}{dt}+\frac{I_1}{C_o}+(L_o+L_2)\frac{d^2I_2}{dt^2}+(R_o+R_2)\frac{dI_2}{dt}\\ &\qquad+\frac{I_2}{C_o}=0,\\ &(L_o+L_1)\frac{d^2I_1}{dt^2}+(R_o+R_1)\frac{dI_1}{dt}+\left(\frac{1}{C_o}+\frac{1}{C_1}\right)I_1+L_o\frac{d^2I_2}{dt^2}\\ &\qquad+R_o\frac{dI_2}{dt}+\frac{I_2}{C_o}=0. \end{aligned}\right\} \tag{15}$$

Assuming as solutions, $I_1=A\varepsilon^{\lambda t}$ and $I_2=B\varepsilon^{\lambda t}$, and elimnating $\frac{A}{B}$ from the above two equations, we get a fourth degree equation in $\lambda$ as follows:

$$\left.\begin{aligned} &(L_o L_1+L_o L_2+L_1 L_2)\lambda^4+(L_oR_1+L_2 R_o+L_2 R_1+L_1 R_o\\ &\quad+L_o R_2+L_1 R_2)\lambda^3+\left(\frac{L_2}{C_o}+\frac{L_o}{C_1}+\frac{L_2}{C_1}+\frac{L_1}{C_o}+R_o R_2\right.\\ &\quad\left.+R_1 R_o+R_1 R_2\right)\lambda^2+\left(\frac{R_2}{C_o}+\frac{R_o}{C_1}+\frac{R_2}{C_1}+\frac{R_1}{C_o}\right)\lambda\\ &\quad+\frac{1}{C_1C_o}=0. \end{aligned}\right\} \tag{16}$$

Substituting for $\lambda$ the complex quantity $\alpha+j\beta$, neglecting $\alpha^2$ in comparison with $\beta^2$, and then separating the real and imaginary terms, we get the following two equations:

$$(L_o L_1+L_o L_2+L_1 L_2)\,\beta^4-\Big\{3\,\alpha\,(L_o R_1+L_2 R_o+L_2 R_1+L_1 R_o$$
$$+L_o R_2+L_1 R_2)+\frac{L_2+L_1}{C_o}+\frac{L_o+L_2}{C_1}+R_o R_2+R_o R_1$$
$$+R_1 R_2\Big\}\beta^2+\left(\frac{R_2+R_1}{C_o}+\frac{R_o+R_2}{C_1}\right)\alpha+\frac{1}{C_1 C_o}=0, \qquad (17)$$

$$4\,(L_o L_1+L_o L_2+L_1 L_2)\,\alpha\,\beta^2+(L_o R_1+L_2 R_o+L_2 R_1$$
$$+L_1 R_o+L_o R_2+L_1 R_2)\,\beta^2-2\left(\frac{L_2+L_1}{C_o}+\frac{L_o+L_2}{C_1}\right.$$
$$\left.+R_o R_2+R_o R_1+R_1 R_2\right)\alpha-\frac{R_2+R_1}{C_o}-\frac{R_o+R_2}{C_1}=0. \qquad (18)$$

Neglecting the resistance terms in equation (17), these being extremely small in comparison with the other terms, it reduces to

$$C_1 C_o\,(L_o L_1+L_o L_2+L_1 L_2)\,\beta^4-(L_2 C_1+L_o C_0+L_2 C_o$$
$$+L_1 C_1)\,\beta^2+1=0, \qquad (19)$$

and

$$\beta^2=\frac{(L_2C_1+L_oC_o+L_2C_o+L_1C_1)\pm\sqrt{(L_2 C_1+L_o C_o+L_2 C_o+L_1 C_1)^2-4\,C_1 C_o\,(L_o L_1+L_o L_2+L_1 L_2)}}{2\,C_1 C_o\,(L_o L_1+L_o L_2+L_1 L_2)} \qquad (20)$$

Equation (20) is the general expression for the two frequency constants of direct coupled circuits.

For the resonance condition,

$$(L_o+L_2)\,C_o=(L_1+L_2)\,C_1=L\,C,$$

equation (20) reduces to

$$\beta^2=\frac{1\pm\sqrt{1-\left(\dfrac{L_o}{L_o+L_2}+\dfrac{L_1 L_2}{(L_o+L_2)\,(L_1+L_2)}\right.}}{LC\left[\dfrac{L_o}{L_o+L_2}+\dfrac{L_1 L_2}{(L_o+L_2)(L_1+L_2)}\right]}. \qquad (21)$$

Put, for brevity,

$$\rho^2=1-\frac{L_o}{L_o+L_2}-\frac{L_1 L_2}{(L_o+L_2)\,(L_1+L_2)},$$
$$=\frac{{L_2}^2}{(L_o+L_2)(L_1+L_2)}, \qquad (22)$$

and substitute in equation (21), we get

$$\beta^2=\frac{1\pm\rho}{L\,C\,(1-\rho^2)},$$
$$=\frac{1}{LC(1\mp\rho)}. \qquad (23)$$

The expression for the frequency constants is identical in form to the one obtained for the magnetically-coupled circuits.

and the expression for $\rho$ given by (22) may be defined as the coefficient of coupling in direct-coupled circuits.

From equation (18), we get the value of $\alpha$, the damping factor, which is as follows:

$$\alpha = \frac{\begin{array}{c} C_o C_1 (L_o R_1 + L_2 R_o + L_2 R_1 + L_1 R_o + L_o R_2 + L_1 R_2)\beta^2 \\ -(R_2 C_1 + R_o C_o + R_2 C_o + R_1 C_1) \end{array}}{\begin{array}{c} 2 (L_2 C_1 + L_o C_o + L_2 C_o + L_1 C_1 + R_o R_2 C_o C_1 + R_o R_1 C_o C_1 \\ + R_1 R_2 C_o C_1) - 4 C_o C_1 \beta^2 (L_o L_1 + L_o L_2 + L_1 L_2) \end{array}}. \tag{24}$$

Equation (24) is the complete expression for the damping factors. Knowing the values of $\beta^2$ as given by (21) or (23), the values of $\alpha$ can be readily evaluated for any given case. For resonance condition, and neglecting the terms

$$R_o R_2 C_o C_1 + R_o R_1 C_o C_1 + R_1 R_2 C_o C_o$$

as being very small in comparison with $LC$, equation (24) reduces to:

$$\alpha = \frac{\begin{array}{c} L^2 C^2 \beta^2 \left\{ \dfrac{R_1}{L_1 + L_2} + \dfrac{R_o}{L_o + L_2} + \dfrac{R_2 (L_1 + L_o)}{(L_o + L_2)(L_1 + L_2)} \right\} \\ -(R_2 C_1 + R_1 C_1 + R_o C_o + R_2 C_o) \end{array}}{4 L C \{1 - L C \beta^2 (1 - \rho^2)\}}. \tag{25}$$

Substituting the value of $\beta^2$ from (23), the above equation reduces to

$$\begin{aligned} \alpha = & \frac{1}{4 (1 \mp \rho)(L_1 + L_2) \rho} \left\{ R_1 + R_o \frac{L_1 + L_2}{L_o + L_2} + R_2 \frac{L_1 + L_o}{L_o + L_2} \right\} \\ & \pm \frac{1}{4\rho} \left\{ \frac{R_2 + R_1}{L_1 + L_2} + \frac{R_o + R_2}{L_o + L_2} \right\}, \end{aligned} \tag{26}$$

which is the complete expression for the damping factors at resonance condition. In all practical cases, $L_2$ is small in comparison with either $L_0$ or $L_1$, and by neglecting $L_2$, equation (26) simplifies to the following:

$$\begin{aligned} \alpha &= \frac{1}{\mp 4 \rho L_1 (1 \mp \rho)} \left\{ R_1 + R_o \frac{L_1}{L_o} \right\} \pm \frac{1}{\rho} \left\{ \frac{R_1}{L_1} + \frac{R_o}{L_o} \right\} \\ &= - \frac{R_1 + R_o \dfrac{L_1}{L_o}}{4 L_1 (1 \mp \rho)} \\ &= - \frac{\dfrac{R_1}{4 L_1} + \dfrac{R_o}{4 L_o}}{1 \mp \rho}. \end{aligned} \tag{27}$$

The expression for the damping factors given by (27) is identical in form to the expression obtained for the case of magnetically coupled circuits.

ELECTROSTATIC COUPLING

The circuit arrangement is shown in Figure 3, and the circuit equations are as follows:

$$\left.\begin{aligned} &L_o\frac{d^2I_o}{dt^2}+R_o\frac{dI_o}{dt}+\frac{I_o}{C_o}+\frac{I_1}{C_1}=0,\\ &L_o\frac{d^2I_o}{dt^2}+R_o\frac{dI_o}{dt}+\frac{I_o}{C_o}+\frac{I'}{C'}+\frac{I_2}{C_2}=0,\\ &L_o\frac{d^2I_o}{dt^2}+R_o\frac{dI_o}{dt}+\frac{I_o}{C_o}+\frac{I'}{C'}+L_2\frac{d^2I_3}{dt^2}+R_2\frac{dI_3}{dt}=0. \end{aligned}\right\} \quad (28)$$

We also have the auxiliary current relations,

$$\left.\begin{aligned} &I'=I_2+I_3,\\ &I_o=I'+I_1=I_1+I_2+I_3. \end{aligned}\right\} \quad (29)$$

Assuming

$$I_1=A\,\varepsilon^{\lambda t},\ I_2=B\,\varepsilon^{\lambda t},\ I_3=D\,\varepsilon^{\lambda t},$$

and also substituting the relations given by (29), equations (28) take the following form:

$$\left.\begin{aligned} &\left(L_o\lambda^2+R_o\lambda+\frac{1}{C_o}+\frac{1}{C_1}\right)A+\left(L_o\lambda^2+R_o\lambda+\frac{1}{C_o}\right)B+\left(L_o\lambda^2\right.\\ &\qquad\left.+R_o\lambda+\frac{1}{C_o}\right)D=0,\\ &\left(L_o\lambda^2+R_o\lambda+\frac{1}{C_o}\right)A+\left(L_o\lambda^2+R_o\lambda+\frac{1}{C_o}+\frac{1}{C'}+\frac{1}{C_2}\right)B+\left(L_o\lambda^2\right.\\ &\qquad\left.+R_o\lambda+\frac{1}{C_o}+\frac{1}{C'}\right)D=0,\\ &\left(L_o\lambda^2+R_o\lambda+\frac{1}{C_o}\right)A+\left(L_o\lambda^2+R_o\lambda+\frac{1}{C_o}+\frac{1}{C'}\right)B+\left(L_o\lambda^2\right.\\ &\qquad\left.+R_o\lambda+\frac{1}{C_o}+L_2\lambda^2+R_2\lambda+\frac{1}{C'}\right)D=0. \end{aligned}\right\} \quad (30)$$

Eliminating $A$ from the above three equations, we get

$$\left.\begin{aligned} &\left\{\left(L_o\lambda^2+R_o\lambda+\frac{1}{C_o}\right)^2-\left(L_o\lambda^2+R_o\lambda+\frac{1}{C_o}+\frac{1}{C_1}\right)\left(L_o\lambda^2+R_o\lambda+\frac{1}{C_o}\right.\right.\\ &\qquad\left.\left.+\frac{1}{C'}+\frac{1}{C^2}\right)\right\}B+\left\{\left(L_o\lambda^2+R_o\lambda+\frac{1}{C_o}\right)^2-\left(L_o\lambda^2+R_o\right.\right.\\ &\qquad\left.\left.+\frac{1}{C_o}+\frac{1}{C_1}\right)\left(L_o\lambda^2+R_o\lambda+\frac{1}{C_o}+\frac{1}{C'}\right)\right\}D=0. \qquad (31)\\ &\qquad\frac{1}{C_2}B-(L_2\lambda^2+R_2\lambda)\,D=0. \end{aligned}\right\}$$

Eliminating $\frac{B}{D}$ from the above two equations, we get a fourth degree equation in $\lambda$ as follows:

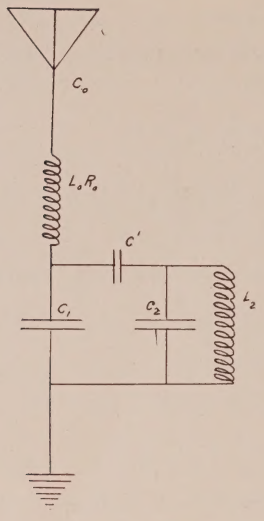


FIGURE 3

$$L_o L_2 (C_1 C_2 + C_1 C' + C' C_2) \lambda^4 + (L_2 R_o + L_o R_2) (C_1 C_2 + C_1 C' + C_2 C') \lambda^3 + \left\{ \left( \frac{L_2}{C_o} + R_o R_2 \right) (C_1 C_2 + C_1 C' + C_2 C') + L_2 (C' + C_2) + L_o (C' + C_1) \right\} \lambda^2 + \left\{ \frac{R_2}{C_o} (C_1 C_2 + C_1 C' + C_2 C') + R_2 (C_2 + C') + R_o (C_1 + C') \right\} \lambda + \frac{C_1}{C_o} + \frac{C'}{C_o} + 1 = 0. \quad (32)$$

Substituting for $\lambda$ the complex quantity $\alpha + j\beta$ and separating reals and imaginaries we get the following two equations:

$$L_o L_2 (C_1 C_2 + C_1 C' + C' C_2) \beta^4 - \left\{ 3\alpha (L_2 R_o + L_o R_2) (C_1 C_2 + C_1 C' + C_2 C') + \left( \frac{L_2}{C_o} + R_o R_2 \right) (C_1 C_2 + C_1 C' + C_2 C') + L_2 (C' + C_2) + L_o (C' + C_1) \right\} \beta^2 + \alpha \left\{ \frac{R_2}{C_o} (C_1 C_2 + C_1 C' + C_2 C') + R_2 (C_2 + C') + R_o (C_1 + C') \right\} + \frac{C_1}{C_o} + \frac{C'}{C_o} + 1 = 0, \quad (33)$$

$$-4\alpha \beta^2 L_o L_2 (C_1 C_2 + C_1 C' + C' C_2) - \beta^2 (L_2 R_o + L_o R_2) (C_1 C_2 + C_1 C' + C_2 C') + 2\alpha \left\{ \left( \frac{L_2}{C_o} + R_o R_2 \right) (C_1 C_2 + C_1 C' + C_2 C') + L_2 (C' + C_2) + L_o (C' + C_1) \right\} + \frac{R_2}{C_o} (C_1 C_2 + C_1 C' + C_2 C') + R_2 (C_2 + C') + R_o (C_1 + C') = 0. \quad (34)$$

Substituting the above value of $L_0$ in (35), we get

$$L_2^2 C_2 (C_o+C_1)\left(\frac{C_2C_1+C_1C'+C_2C'}{C_oC_1}\right)\beta^4 - L_2\left\{\frac{C_2C_1+C_1C'+C_2C'}{C_o}+C_2+C'+\frac{C_2(C_o+C_1)(C_1+C')}{C_oC_1}\right\}\beta^2 + \frac{C_1+C'+C_o}{C_o}=0, \tag{37}$$

which simplifies to the following:

$$L^2C^2\left(C_1+\frac{C_1C'}{C_2}+C'\right)\beta^4 - LC\left(2C_1+\frac{C_1C'}{C_2}+C'+\frac{C_1C'}{C_1+C_o}\right)\beta^2+C_1\left(1+\frac{C'}{C_1+C_o}\right)=0 \tag{38}$$

and

$$\beta^2=\frac{\left(2C_1+\frac{C_1C'}{C_2}+C'+\frac{C_1C'}{C_1+C_o}\right)\pm\sqrt{\left(2C_1+\frac{C_1C'}{C_2}+C'+\frac{C_1C'}{C_1+C_o}\right)^2-4C_1\left(1+\frac{C'}{C_1+C_o}\right)\left(C_1+\frac{C_1C'}{C_2}+C'\right)}}{2LC\left(C_1+\frac{C_1C'}{C_2}+C'\right)}. \tag{39}$$



Expanding the terms under the radical and rearranging, we get

$$\beta^2=\frac{\left(2+\frac{C'}{C_2}+\frac{C'}{C_1}+\frac{C'}{C_1+C_o}\right)\pm\sqrt{\left(\frac{C'}{C_2}+\frac{C'}{C_1}\right)^2-\frac{2C'}{C_1+C_o}\left(\frac{C'}{C_1}+\frac{C'}{C_2}\right)+\frac{C'}{(C_1+C_o)^2}}}{2LC\left(1+\frac{C'}{C_2}+\frac{C'}{C_1}\right)}. \tag{40}$$

Put, for brevity,

$$\left.\begin{aligned}\frac{C'}{C_1}+\frac{C'}{C_2}&=K_1\\ \frac{C'}{C_1+C_o}&=K_2\end{aligned}\right\}, \tag{41}$$

In equation (33) all the resistance terms are very small and can be neglected, and it reduces to the following:

$$L_o L_2 (C_1 C_2 + C_1 C' + C_2 C') \beta^4 - \left\{ \frac{L_2}{C_o} (C_1 C_2 + C_1 C' + C_2 C') + L_2 (C' + C_2) + L_o (C' + C_1) \right\} \beta^2 + \frac{C_1 + C'}{C_o} + 1 = 0. \tag{35}$$

The value of $\beta$ can be readily obtained by solving the above biquadratic equation. The formula obtained, however, will be difficult of interpretation because of its complex form. It is, therefore, desirable in this case to limit the discussion to the resonance condition. If it should be necessary in some cases to obtain the values of the frequency constants for the non-resonance condition, it would be simpler to substitute the numerical values for the different $L$'s and $C$'s in (35) and solving for $\beta$.

For resonance, we have

$$L_2 C_2 = L_o \frac{1}{\frac{1}{C_o} + \frac{1}{C_1}} = \frac{L_o C_o C_1}{C_o + C_1} = LC. \tag{36}$$

$$L_o = \frac{L_2 C_2 (C_o + C_1)}{C_o C_1}.$$

and substitute in (40). We finally get the following expression for the frequency constants:

$$\beta^2 = \frac{(2+K_1+K_2) \pm \sqrt{K_1^2 - 2K_1K_2 + K_2^2}}{2LC(1+K_1)},$$

$$= \frac{2+(K_1+K_2) \pm (K_1-K_2)}{2LC(1+K_1)}. \tag{42}$$

The damping factors of the system can be obtained from equation (34).

$$\alpha = \frac{(L_2R_o + L_oR_2)(C_1C_2 + C_1C' + C_2C')\beta^2 - \frac{R_2}{C_o}(C_1C_2 + C_1C' + C_2C') - R_2(C_2+C') - R_o(C_1+C')}{2\left\{\left(\frac{L_2}{C_o} + R_oR_2\right)(C_2C_1 + C_1C' + C_2C') + L_2(C_2+C') + L_o(C_1+C')\right\} - 4L_oL_2(C_1C_2 + C_1C' + C_2C')\beta^2}. \tag{43}$$



If the values of $\beta^2$ and the electrical constants of the circuits are given, the values of the damping factors are completely determined by the above equation.

For resonance condition

$$L_o = \frac{L_2C_2(C_1+C_o)}{C_oC_1},$$

and also neglecting $R_oR_2$ in comparison with $\frac{L_2}{C_o}$, equation (43) takes the form:

$$\alpha = \frac{L_2R_o(C_2C_1 + C_1C' + C_2C')\beta^2 + L_2R_2C_2\frac{(C_o+C_1)}{C_oC_1}(C_2C_1 + C_1C' + C_2C')\beta^2 - R_2\left(\frac{C_1C_2 + C_1C' + C_2C'}{C_o} + C_2 + C'\right) - R_o(C_1+C')}{2L_2\left\{\frac{C_2C_1 + C_1C' + C_2C'}{C_o} + C_2 + C' + \frac{C_2(C_o+C_1)}{C_oC_1}(C_1+C')\right\} - 4L_2^2\frac{C_2(C_o+C_1)}{C_oC_1}(C_2C_1 + C_1C' + C_2C')\beta^2}. \tag{44}$$

The above equation may be put in the following form:

$$\alpha = \frac{L C R_o \beta^2 (1+K_1) + L C R_2 \beta^2 \frac{C' C_2}{K_2 C_o C_1} (1+K_1) - R_2 \left\{ (C_2+C') \left( \frac{1}{C_o} + \frac{1}{C_1} \right) + \frac{C' C_2}{C_1 C_o} \right\} - R_o \left( 1+\frac{C'}{C_1} \right)}{2 L_2 \left\{ (C_2+C') \left( \frac{1}{C_o} + \frac{1}{C_1} \right) + \frac{C_2 C'}{C_1 C_o} + C_2 \left( \frac{1}{C_1} + \frac{1}{C_o} \right) \left( 1+\frac{C'}{C_1} \right) \right\} - 4 L^2 C^2 \beta^2 \frac{C' (1+K_1)}{K_2 C_o C_1}}. \tag{45}$$

Putting

$$\frac{1}{C_a} = \frac{1}{C_o} + \frac{1}{C_o},$$

where $C_a$ is the series antenna capacity, equation (45) reduces to

$$\alpha = \frac{L C \beta^2 (1+K_1) \left( R_o + R_2 \frac{C_2}{C_a} \right) - R_2 \left( \frac{C_2+C_1}{C_a} + \frac{C' C_2}{C_1 C_o} \right) - R_o \left( 1+\frac{C'}{C_1} \right)}{2 L_2 \left\{ \frac{2 C_2+C'}{C_a} + \frac{C_2 C'}{C_1 C_o} + \frac{C_2 C'}{C_1 C_a} \right\} - 4 L^2 \frac{C^2}{C_a} \beta^2 (1+K_1)}. \tag{46}$$

Introducing the values of $\beta^2$ as given by equation (42),

$$\beta_1^2 = \frac{1}{L C},$$

$$\beta_2^2 = \frac{1}{L C} \frac{1+K_2}{1+K_1},$$

and performing the various algebraic transformations, we arrive at the following formula for the damping factors:

$$\left.\begin{aligned} \alpha_1 &= -\frac{R_o + R_2 \frac{C_2^2}{C_1^2}}{2 L_2 \left( \frac{C_2^2}{C_1^2} + \frac{C_2}{C_a} \right)} . \\ \alpha_2 &= -\frac{R_o \frac{C'}{C_1} \left( 1 - \frac{C_a}{C_o} \right) + R_2 \frac{C'}{C_a}}{2 L_2 \left\{ \frac{C'}{C_a} + \frac{C' C_2}{C_1^2} \right\}} . \end{aligned} \right\} \quad (47)$$

SUMMARY: The general theory of coupled radio frequency circuits is developed, for systems having two degrees of freedom, and for the resonance and non-resonance conditions. The theory is applied to determine the frequencies and damping factors for inductively coupled, directly coupled, and electrostatically coupled circuits.

# DIGEST OF UNITED STATES PATENTS RELATING TO RADIO TELEGRAPHY AND TELEPHONY*

GRANTED JANUARY 4, 1921–FEBRUARY 15, 1921

BY

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The object of this section of the PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS is to make available in convenient form for research engineers and others interested, brief information on the radio patents which are granted each week by the Patent Office. The rapid developments in this art emphasize the importance of radio research engineers being familiar with patent literature in order to eliminate as far as possible the duplication of research effort. It is not the purpose of this section to explain radio inventions fully, but merely to indicate the general nature of the patents in order that those of particular interest to individuals concerned with current problems may be selected, and copies of the patents obtained for complete study. Copies of the complete patents may be obtained at ten cents each by addressing the Commissioner of Patents at Washington, D. C.

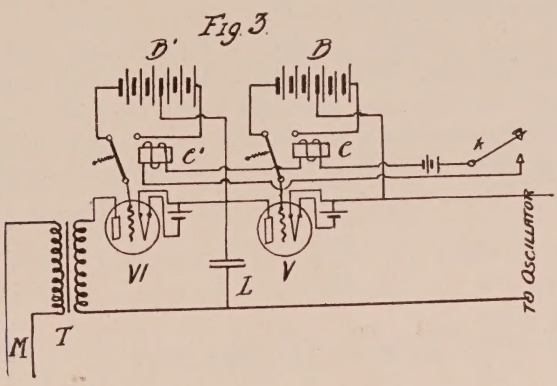
1,364,495—Henry Joseph Round, of London, and William Theodore Ditchman, of Twichenham, England, Assignors to Radio Corporation of America, of New York, N. Y., a corporation of Delaware.

Wireless Telegraph-Transmitter—Patented January 4, 1921.

This invention shows a circuit wherein current from an alternating source $M$ is rectified and then smoothed out by a condenser $L$ and utilized as a source of direct current for a valve oscillator. In the diagram, the grids of the tubes $V$ and $V'$ are connected to the armatures of two relays $C$ and $C'$ arranged in circuit with a key $K$, the opening and closing of which causes the

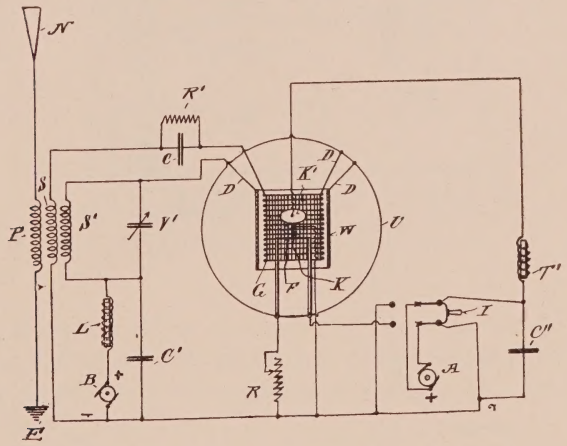
*Received by the Editor, March 3, 1921. While great care has been taken in the preparation of these Digests, THE INSTITUTE OF RADIO ENGINEERS assumes no responsibility for their correctness or completeness, or for possible omissions of particular patents.—EDITOR.

energization of the relays, whereby the grids of the two tubes are simultaneously connected to corresponding ends of batteries $B\ B'$ and are thereby made positive and negative with respect to the filaments whereby the circuits on the two sides of condenser $L$ are opened and closed.




NUMBER 1,364,495—Wireless Telegraph Transmitter

1,365,157—Lee de Forest, of New York, N. Y., Assignor to de Forest Radio Telephone and Telegraph Company, of New York., a corporation of Delaware.
Apparatus for Use in Telegraphy or Telephony—Patented January 11, 1921.




NUMBER 1,365,157—Apparatus for Use in Telegraphy or Telephony

This invention pertains to a construction of audion in which an electric arc replaces the usual filament electrode.

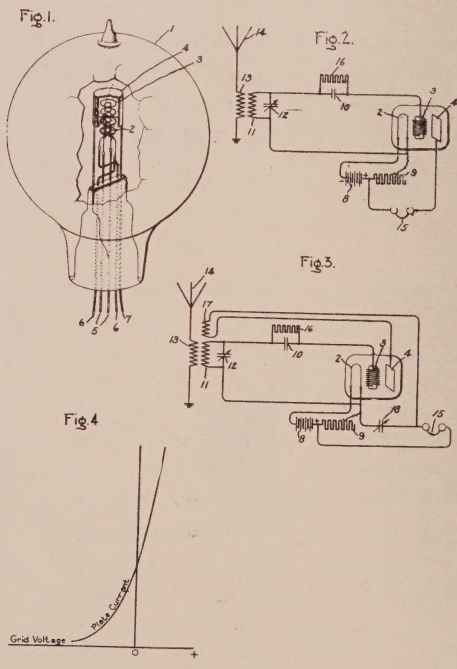
1,365,292—Philip Thomas, of Edgewood Park, Pennsylvania, Assignor to Westinghouse Electric and Manufacturing Co., a corporation of Pennsylvania.

Method of Making Condensers—Patented January, 11, 1921.

This patent covers a method of impregnating condensers. The process comprises immersing the condensers in a solution of benzol and Montan wax, removing them after impregnation, immersing the condensers in a bath of commercially pure Montan wax at about 160 degrees C., then removing the condensers, and subjecting them to pressure in a cold press.

1,365,576—William C. White, of Schenectady, New York, Assignor to General Electric Company, a corporation of New York.

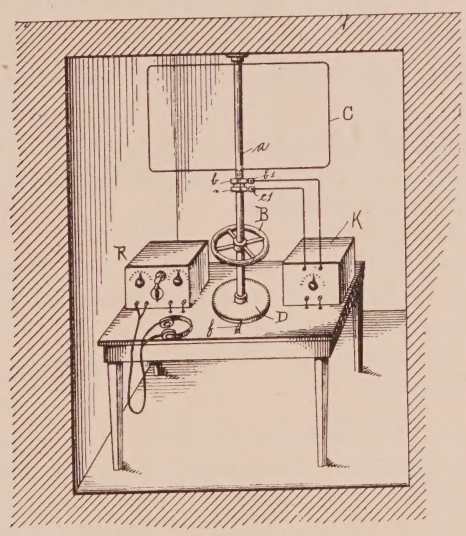
Radio-receiving System—Patented January 11, 1921.




NUMBER 1,365,576—Radic-Receiving System

This patent shows an electron discharge tube for which a single battery is employed to function as a supply for the cathode anode circuit, the filament heating source, and also as a source between the cathode and grid for adjusting the normal potential of the grid. The construction of the tube is such that the cathode and grid are separated by as small a distance as possible and the anode spaced from the grid by a distance which may be slightly greater than the spacing between cathode and grid but which will in any case be very small.

1,365,579—Thomas Appleby and Lloyd M. Knoll, of Philadelphia, Pennsylvania, Assignors of one-third to Cornelius D. Ehret, of Philadelphia, Pennsylvania.
Radio apparatus—Patented January 11, 1921.

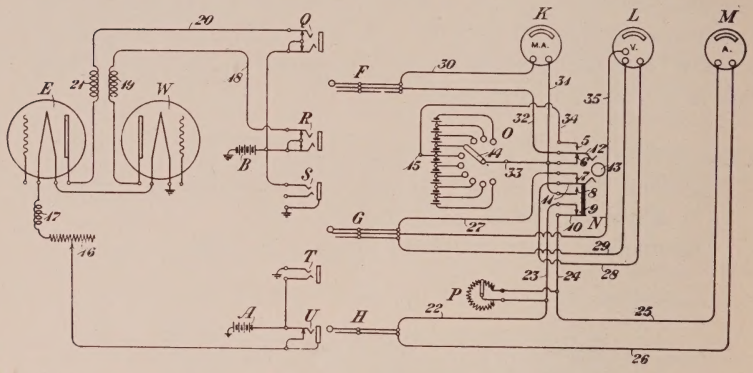



Number 1,365,579—Radio Apparatus

This patent shows a rotary loop antenna arranged underground and shielded to reduce distortional effects. Other figures of the patent show arrangements of shielded loops for multiplex operation.

1,365,734—Samuel P. Shackleton, of New York, N. Y., Assignor to American Telephone and Telegraph Company, a corporation of New York.

Electron-Tube-Testing Circuits—Patented January 18, 1921.




Number 1,365,734—Electron-Tube-Testing Circuits

This patent pertains to a testing apparatus for vacuum tubes to determine expeditiously whether or not the tube is operating within certain prescribed limits.

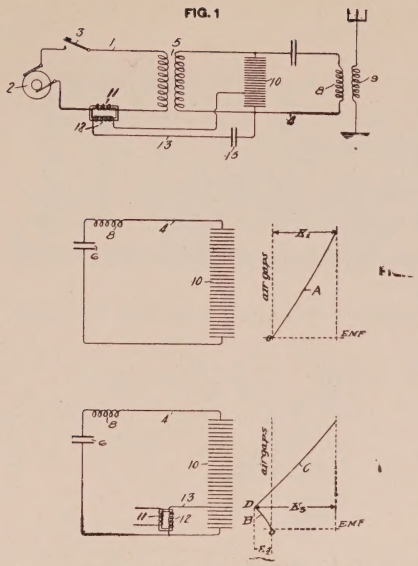
1,365,787—Fred H. Kroger, of Brooklyn, New York, Assignor by Mesne Assignments, to International Radio Telegraph Company, a corporation of Delaware.

Method and Apparatus for Wireless Signaling—Patented January 18, 1921.

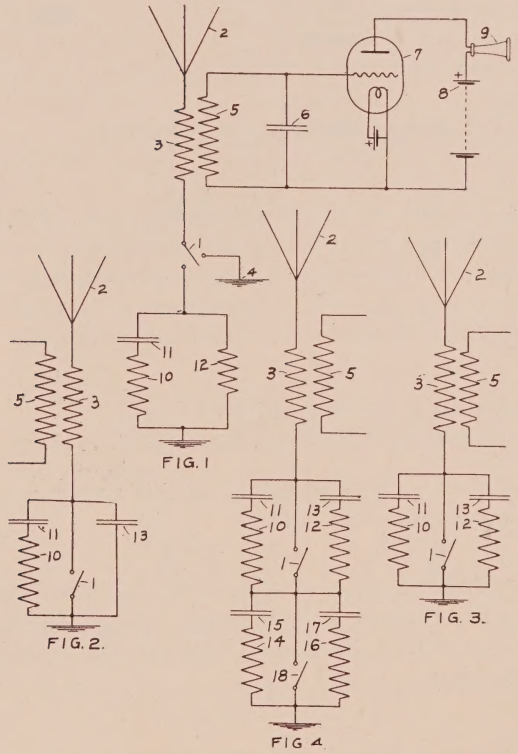
The object of this invention is to secure a more perfect tone control of a quenched gap transmitter. The transmitter circuit is arranged as shown in the diagram with a step-up transformer having its primary winding 11 in the power circuits and its secondary 12 in a control circuit 13 connected across part of the gap. The function of the auxiliary circuit is to impress on the gap an audio frequency impulse at a definite instant in each half cycle of the normal voltage whereby the gap breaks down at regular intervals.

1,365,926—Tyng M. Libby, of Tacoma, Washington, Assignor to Henry G. Cordes, of Bremertown, Washington.

Radio Interference Preventer—Patented January 18, 1921.




NUMBER 1,365,787—Method and Apparatus for Wireless Signaling




NUMBER 1,365,926—Radio Interference Preventer

This invention relates to a receiving circuit so arranged that interfering signals can be counteracted and signals of a desired frequency received.

1,365,977—Leonard F. Fuller, of San Francisco, California, Assignor, by Mesne Assignments, to the United States of America.
Radiotelegraphy—Patented January 18, 1921.

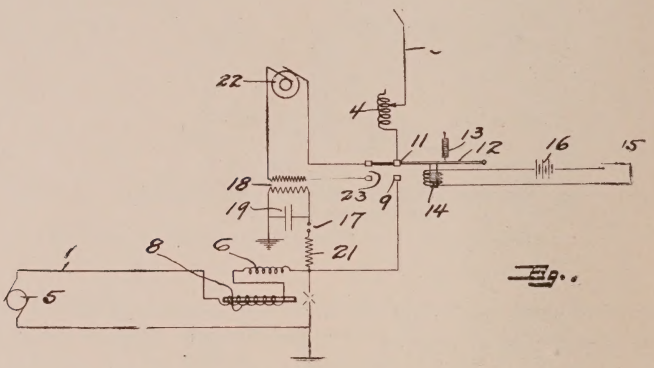



Fig. 1.

Number 1,365,977—Radiotelegraphy

This patent relates to an arc signaling system. The antenna circuit is opened and closed in accordance with signal characters to be transmitted. The arc is extinguished each time that the antenna circuit is opened, but re-ignited upon the closing of the antenna circuit by energy from an auxiliary source 22.

1,366,160—August J. Kloneck, of New York, N.Y.
Rotary Spark-Gap—Patented January 18, 1921.

This patent concerns a particular circuit arrangement for a rotary spark-gap radio transmitter.

1,366,311—Fulton Cutting, of Tuxedo Park, New York, and Bowden Washington, of Cambridge, Massachusetts.
Production of High-Frequency Oscillations—Patented January 18, 1921.

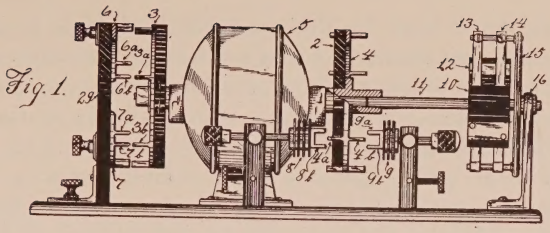


Fig. 1.

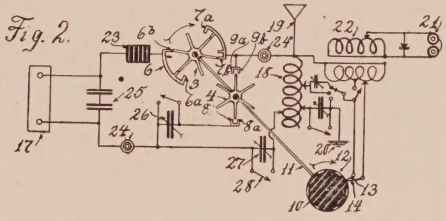


Fig. 2.

NUMBER 1,366,160—Rotary Spark-Gap

Fig. 1.

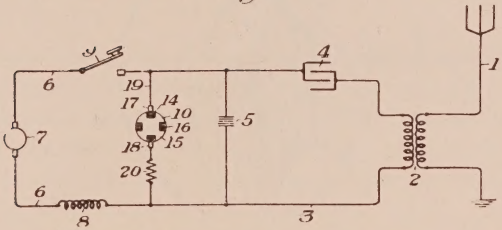
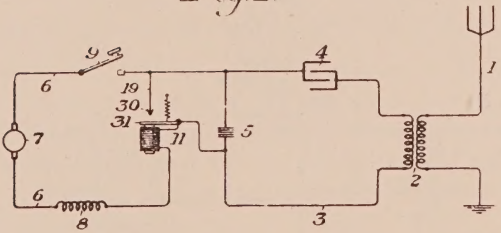


Fig. 2.



NUMBER 1,366,311—Production of High-Frequency Oscillations

This patent relates to the production of high frequency oscillations by periodically exciting a discharge gap and condenser circuit so as to produce groups of radio frequency oscillations. The circuit includes a condenser, a supply circuit, a discharge gap for the condenser, of for example the Chaffee type, and a make and break which operates at an audible frequency when the oscillations are being produced to short circuit the gap periodically, whereby the radio frequency oscillations have a group frequency corresponding to that of the make and break device.

1,366,411—Alexander McLean Nicolson, of New York, N. Y., Assignor to Western Electric Comapny, Incorporated, of New York, N. Y., a corporation of New York.
Thermionic Translating Device—Patented January 25, 1921

Fig. 1

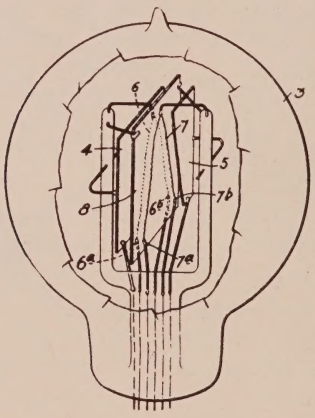
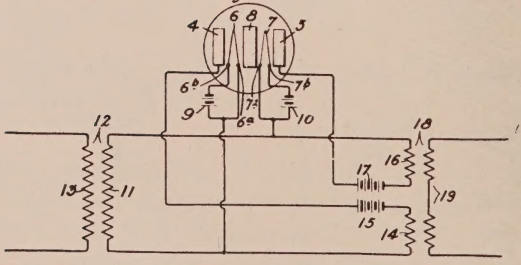


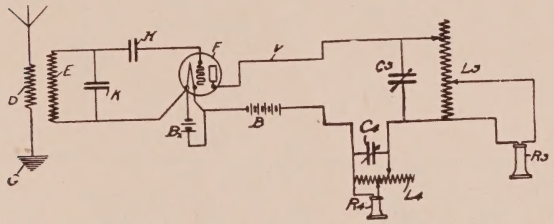

Fig. 2.




Number 1,366,411—Thermionic Translating Device

This patent shows a vacuum tube comprising two cathodes. a dielectric sheet of mica separating the cathodes and an anode, adjacent to each of the cathodes. The tube is arranged in circuit in such manner that one cathode always serves as a control electrode for the electron flow from the other cathode.

1,366,830—Frederick E. Pernot, of Berkeley, California, Assignor of one-third to George Lothaine Greves, of Berkeley, California.
Frequency-Selecting Receiving-Circuit—Patented January 25, 1921.

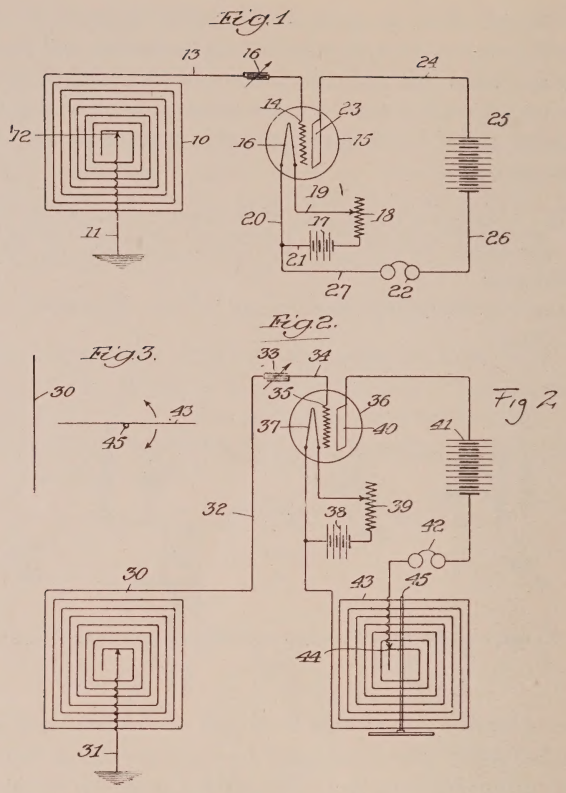



Number 1,366,830—Frequency-Selecting Receiving Circuit

This invention relates to a receiving circuit arranged to segregate and respond simultaneously to separate signal impulses of different group frequencies. The circuit is shown in the accompanying diagram wherein two operators receive simultaneously signals of different group frequencies, one by means of circuit $C_3L_3R_3$ and the other by aid of circuit $C_4L_4R_4$. Each receiver circuit is sensitive to one group frequency only.

1,366,953—Henry K. Sandell, of Chicago, Illinois, Assignor to Herbert S. Mills, of Chicago, Illinois.
Radio Receiving Apparatus—Patented February 1, 1921.

A form of receiving circuit is shown in this patent. A grounded loop antenna 30 is employed connected at the other end with the grid of a vacuum tube. In another form a second loop antenna in proximity to the first grounded antenna is connected in the plate circuit and arranged for angular adjustment relative to the first loop.

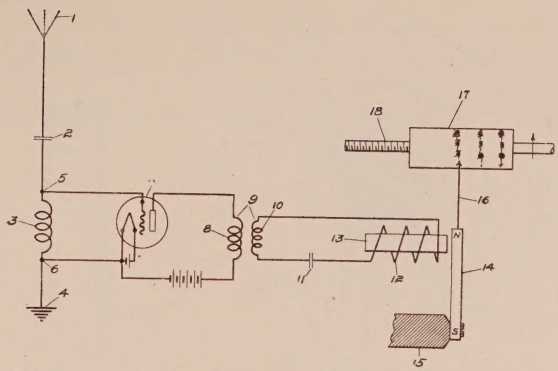



NUMBER 1,366,953—Radio-Receiving Apparatus

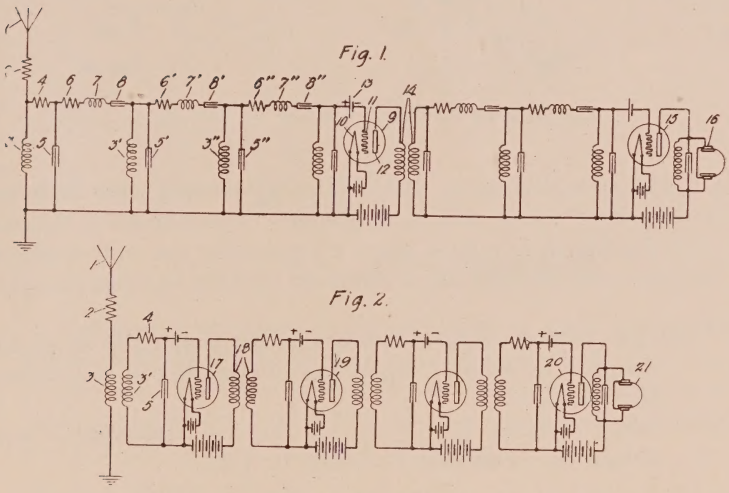
1,367,165—Haraden Pratt, of San Francisco, California. Radio Telegraphic Receiving System—Patented February 1, 1921.

This patentee claims a receiving system having a tuned circuit containing an electromagnet with armature mechanically tuned and adapted to vibrate in synchronism with the radio frequency energy received by the antenna system. The armature is shown associated with tracing arm of an inking recorder.

1,367,224—Harold de Forest Arnold, of East Orange, New Jersey, Assignor to Western Electric Company, Incorporated, of New York, N. Y., a corporation of New York. Radio-Receiving System—Patented February 1, 1921.



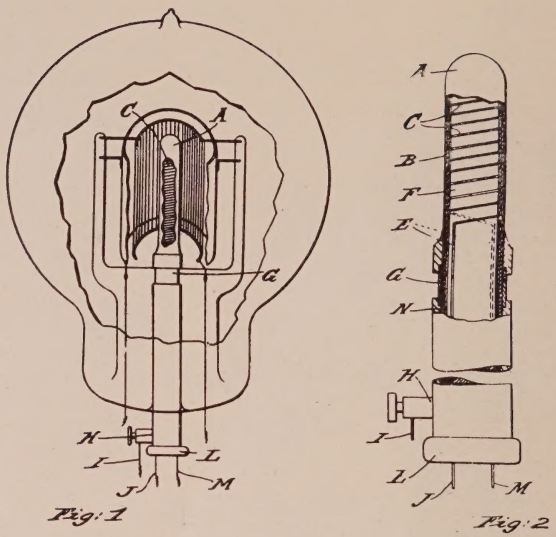
NUMBER 1,367,165—Radio Telegraphic Receiving System




NUMBER 1,367,224—Radio-Receiving System

This invention pertains to a highly damped receiving circuit selectively responsive to signal oscillations of a given frequency. The curcuit includes an antenna system, and a chain of resonant circuits coupled to the antenna, each circuit of the chain being rendered aperiodic by damping.

1,368,584—Samuel S. Torrisi, of Philadelphia, Pennsylvania. Cathode for Audions—Patented February 15, 1921.

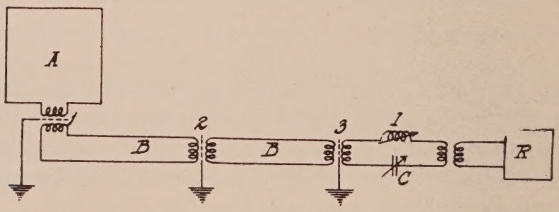



NUMBER 1,368,584—Cathode for Audions

This patent shows a construction of vacuum tube designed to eliminate the filamentary cathode. The cathode is formed of a heating coil wound upon a removable rod and enclosed within a cathode tube which forms the electron emitting element.

1,368,622—Charles Samuel Franklin, of Buckhurst Hill, England, Assignor to Radio Corporation of America, a corporation of Delaware.

Aerial System Employed in Wireless Telegraphy and Telephony—Patented February 15, 1921.

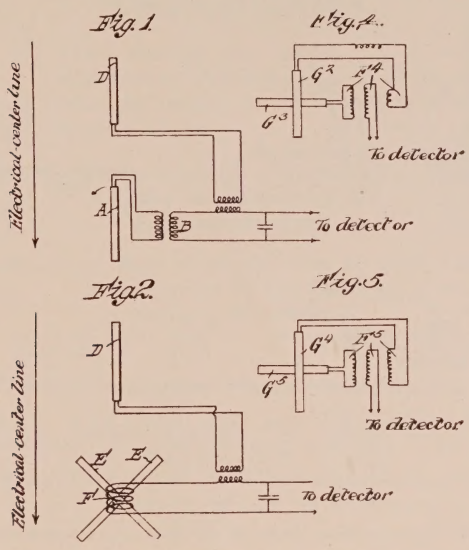



NUMBER 1,368,622—Aerial System Employed in Wireless Telegraphy and Telephony

This invention pertains to a system for loop radio reception.

1,368,657—Henry Joseph Round, of Muswell Hill, London, England, Assignor to Radio Corporation of America, of New York, N. Y., a corporation of Delaware.

Wireless Direction-Finder—Patented February 15, 1921.




NUMBER 1,368,657—Wireless Direction-Finder

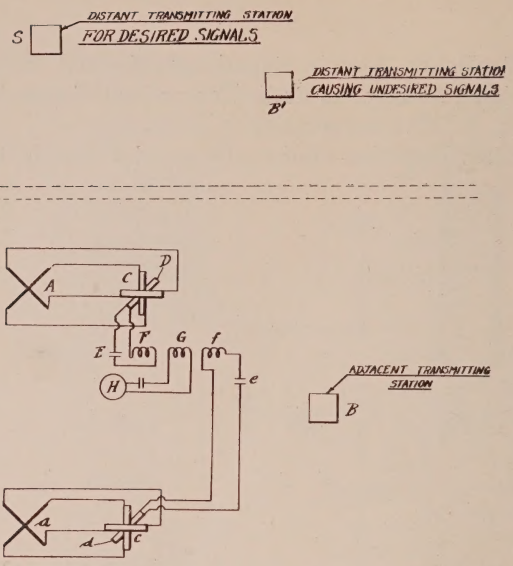
This invention relates to the reduction of error in direction-finder readings where the receiving direction-finder antenna is surrounded by interfering metallic structures such as parts of a ship or aeroplane. The effect of these structures is compensated to give the true direction of signals.

Reissue 15,040—Charles S. Franklin, of London, England, Assignor, by Mesne Assignments, to Radio Corporation of America, a corporation of Delaware.

Aerial Conductor for Wireless Telegraphy—Reissued February 15, 1921.

This patent relates to a reciving circuit for duplex operation in conjunction with an adjacent transmitter. The circuits are based on the principle that antennas of two loops placed at right angles to each other and used in connection with a radio-goniometer can receive best from any two opposite directions and eliminate signals from any two opposite directions at right angles to the first.

Fig. 4.

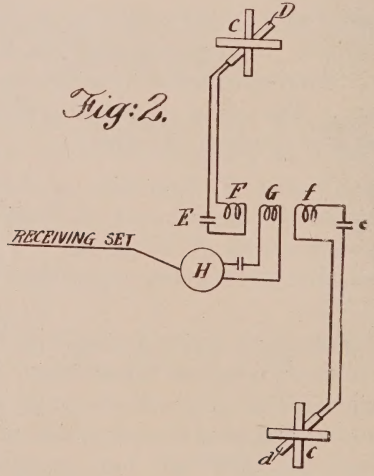



REISSUE NUMBER 15,040—Aerial Conductor for Wireless Telegraphy

Reissue 15,041—Charles S. Franklin, of London, England, Assignor, by Mesne Assignments, to Radio Corporation of America, a corporation of Delaware.

Aerial Conductor for Wireless Telegraphy—Reissued February 15, 1921.

Fig. 2.




REISSUE NUMBER 15,041—Aerial Conductor for Wireless Telegraphy

This patent relates to an antenna system for a receiving station adapted to operate in conjunction with an adjacent transmitting station for duplex radio telegraphy.

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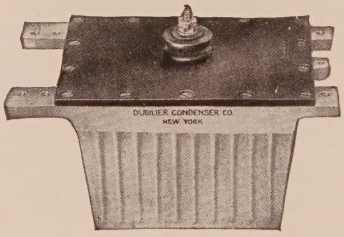


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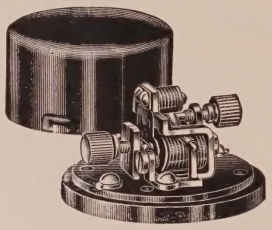

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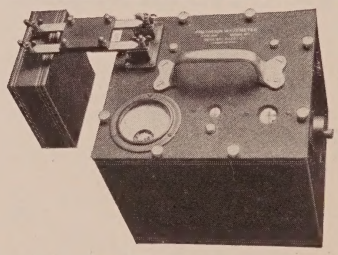

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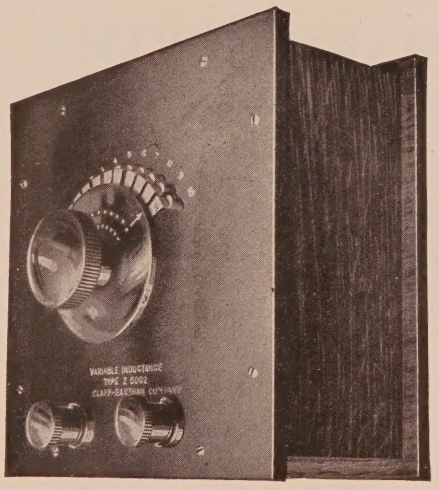

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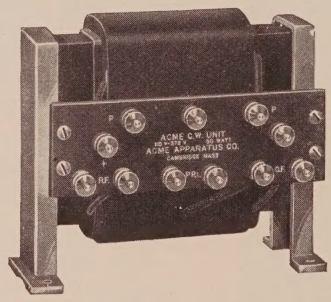


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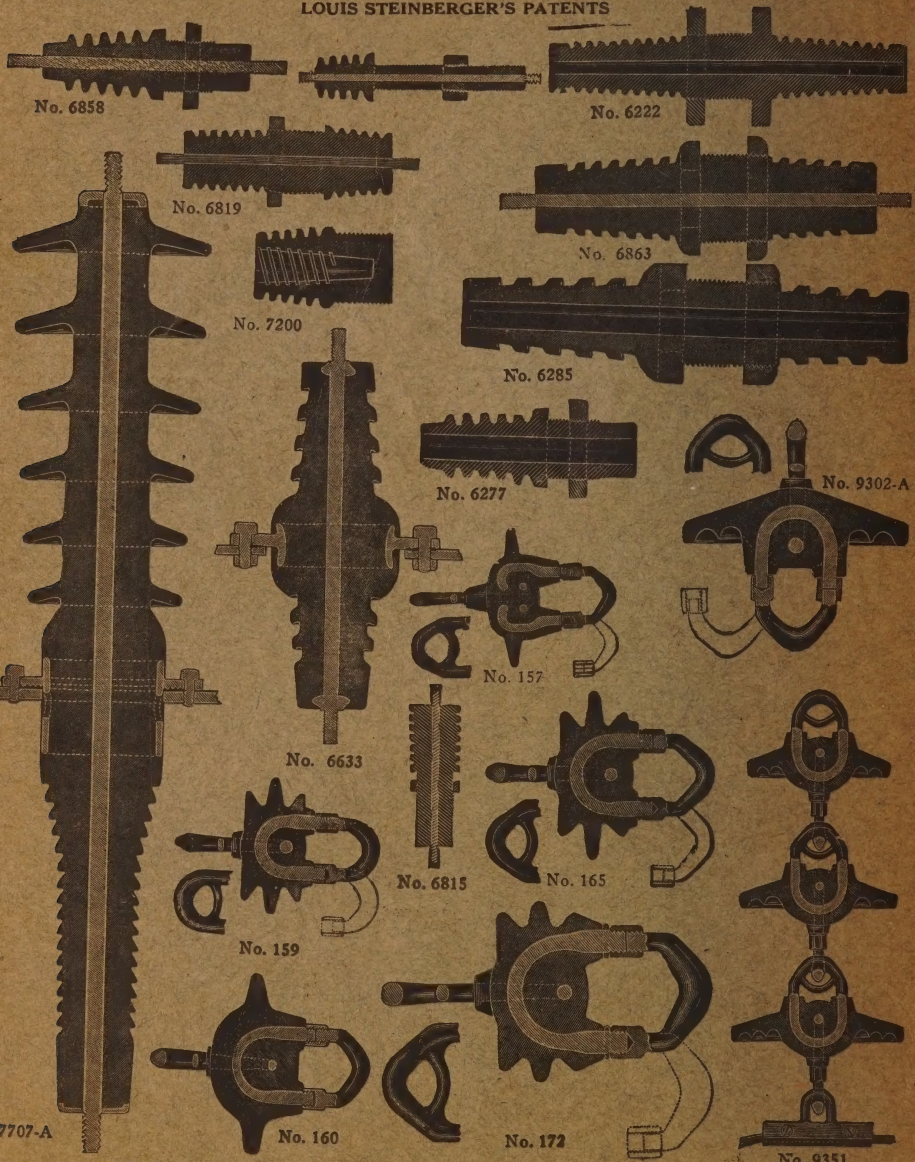


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